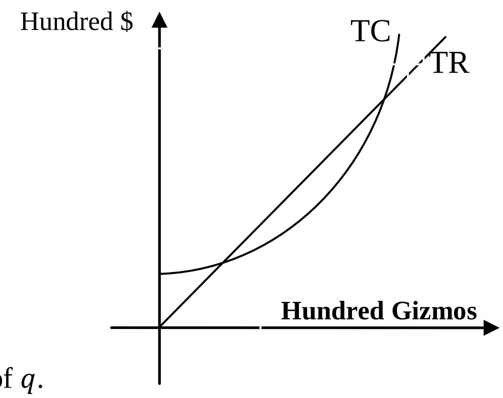
1. (20 pts) The graphs to the right are of the total cost TC and total revenue TR for producing and selling Gizmos. The formula for the total cost is:

$$TC(q) = q^2 + 4.5q + 5$$

with q in **hundreds** of Gizmos, and TC in hundreds of dollars. The TR graph is a line that goes through the origin and crosses the graph of TC at $q = 10$ hundred Gizmos.



a) (4 pts) Write down a formula for the total revenue as a function of q.

Since TR crosses TC at q=10, we know that $TR(10) = TC(10) = 10^2 + 4.5 \cdot 10 + 5 = 150.$

So the slope of TR is $\frac{150-0}{10-0} = 15$. *It's a line through origin, so its y-intercept is zero.*

$TR(q) =$ ___$15q$_____

b) (6 pts) What is the smallest quantity q at which the average cost is \$10 per Gizmo? (round your answer to 4 decimal digits)

The formula for the average cost is: $AC(q) = \frac{TC(q)}{q} = \frac{q^2+4.5q+5}{q} \left(in\ \frac{100's\ \$}{100's\ Gizmos} = \frac{\$}{Gizmo}\right)$. *We are asked for the smallest value of q at which the average cost is \$10, so set AC(q)=10, and solve for q.*

$$\frac{q^2 + 4.5q + 5}{q} = 10$$

Multiplying both sides by q, the denominator cancels and we get a quadratic equation:

$$\frac{q^2 + 4.5q + 5}{\cancel{q}} \cancel{q} = 10q$$

$$q^2 + 4.5q + 5 = 10q$$

$$q^2 - 5.5q + 5 = 0$$

Applying the quadratic formula:

$$q = \frac{5.5 \pm \sqrt{(-5.5)^\wedge 2 - 4(1)(5)}}{2} = \frac{5.5 \pm \sqrt{10.25}}{2} = 1.1492 \text{ or } 4.3507$$

The smallest value is 1.1492.

ANSWER: at $q =$ ___1.1492___ hundred Gizmos.

c) (4 pts) What is the marginal cost at 3 hundred Gizmos? (caution: q is in **hundreds** of Gizmos!)

$$MC(3) = TC(3.01) - TC(3) = [(3.01)^2 + 4.5(3.01) + 5] - [3^2 + 4.5 \cdot 3 + 5] = 27.6051 - 27.5 = 0.1051$$

ANSWER: $MC(3) =$__0.1051_ hundred dollars=\$10.51

d) (6 pts) Compute the largest profit possible.

$$\text{Profit}(q) = \text{TR}(q) - \text{TC}(q) = 15q - (q^2 + 4.5q + 5) = -q^2 + 10.5q - 5$$

Since the profit is a quadratic function, whose graph is a concave-down parabola, it will be maximal at its vertex: $q = \frac{-10.5}{2(-1)} = 5.25$ hundred Gizomos (no need to round to a whole number quantity since this is a whole number of Gizmos).

$$\text{Profit}(5.25) = -5.25^2 + 10.5 \cdot 5.25 - 5 = 22.5625$$

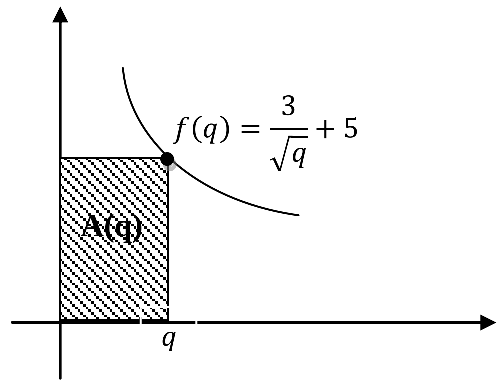
ANSWER: Max profit is __22.5625___ hundred dollars.

2. (12 pts) The graph to the right is of the function

$$f(q) = \frac{3}{\sqrt{q}} + 5$$

We define a new function $A(q)$ by:

$A(q)$ is the area of the rectangle under the graph, with one corner at the point $(q, f(q))$. (see picture)



a) (3 pts) Write out a formula for $A(q)$ in terms of q.

$$A(q) = (base)(height) = qf(q) = \underbrace{q\left(\frac{3}{\sqrt{q}} + 5\right) = \frac{3q}{\sqrt{q}} + 5q = 3\sqrt{q} + 5q}_{either\ of\ these\ three\ would\ be\ OK}$$

ANSWER: $A(q) =$____ $\frac{3q}{\sqrt{q}} + 5q$ ______

b) (6 pts) Find the area of the rectangle at the point on the graph where $f(q) = 6$.

$$f(q) = \frac{3}{\sqrt{q}} + 5 = 6$$

$$\frac{3}{\sqrt{q}} = 1$$

Multiply both sides by $\sqrt{q}$ to get:

$$3 = \sqrt{q}$$

Square both sides to get: $q = 9$

The area at $q = 9$ is $A(9) = 9f(9) = 9 \cdot 6 = 54$

ANSWER: area =_____54______

c) (3 pts) Suppose the function $f(q) = \frac{3}{\sqrt{q}} + 5$ computes the average cost, in dollars per item, for producing q hundred items. In this case, what does the area of the rectangle function compute? What are its units?
(no need to justify your answer)

$$Area = q \cdot AC(q) = TC(q)$$

ANSWER: $A(q)$ would compute the ______Total Cost____ at q hundred items,

in units of: _____hundreds of dollars______

3. (18 points) A car drives for 4 hours on a straight road. Its distance, in miles, from its starting place at t hours is given by the formula:

$$D(t) = 100t - 25t^2$$

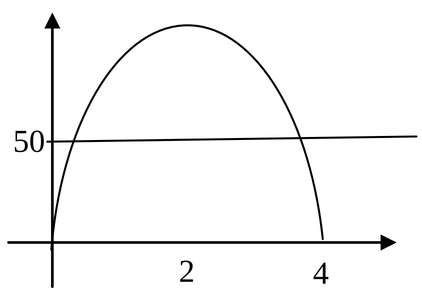
a) (3 pts) At what time is this car the farthest away from its starting place?

The graph of the distance function $D(t) = 100t - 25t^2$ is a concave-down parabola, so it's largest at its vertex:

$$t = -\frac{100}{2(-25)} = 2$$

ANSWER: at_____2______ hours

b) (6 pts) Sketch the graph of $D(t)$ and compute the time interval when this car will be at a distance of at least 50 miles from its starting place.



$D(t) = 100t - 25t^2 = 50$
$-25t^2 + 100t - 50 = 0$

Quadratic formula: $t = 0.58578 \ldots \approx 0.59$, and $t = 3.414213 \ldots \approx 3.41$

From the picture, we see that $D(t) \geq 50$ between the two points:

ANSWER: from $t =$ _0.59__to $t =$ __3.41___hours

c) (5 pts) Write the following expression as a linear function of t:

$$\begin{aligned} D(t+0.5) - D(t) &= [100(t+0.5) - 25(t+0.5)^2] - [100t - 25t^2] \\ &= \cancel{100t} + 50 - 25(t^2 + t + 0.25) - \cancel{100t} + 25t^2 \\ &= 50 - \cancel{25t^2} - 25t - 6.25 + \cancel{25t^2} \\ &= 43.75 - 25t \end{aligned}$$

ANSWER: $D(t + 0.5) - D(t) =$ ____$43.75 - 25t$ _____

d) (4 pts) Find a time t such that the car traveled 25 miles during the half-hour time interval starting at t.

In functional notation: $D(t + 0.5) - D(t) = 25$
Using the formula we found in part c): $43.75 - 25t = 25$

$$-25t = -18.75$$
$$t = 0.75$$

ANSWER: $t =$ ___0.75____ hours