Solutions Version 1

NAME: ________________________________ Student ID #: _________________

QUIZ SECTION:______

Math 111
Midterm II

February 20th, 2007

Problem 1	15	
Problem 2	14	
Problem 3	15	
Problem 4	6	
Total:	**50**	

- Your exam should contain **5 pages in total and 4 problems**. Please check your test for completeness.
- You **must use the methods of this class to solve the problems, and you must show entirely how you get your answers**. Work done "in your head" cannot get credit. Work done by guessing and checking, or by reading off values on a graphing calculator may get little credit. Correct answers with incomplete, wrong or missing work will get partial credit at best.
- Write your final answer in the indicated spaces. Unless otherwise specified, round off your final answer to the nearest two decimal digits.
- If you need more room, use the backs of pages and indicate to the reader that you have done so. If you still need more paper, ask your TA for some more, write your name and section on it and make sure you turn it in to your TA inside your test.
- Read each question carefully. Do not get stuck on any one question for more than a few minutes.
- Raise your hand if you have a question.

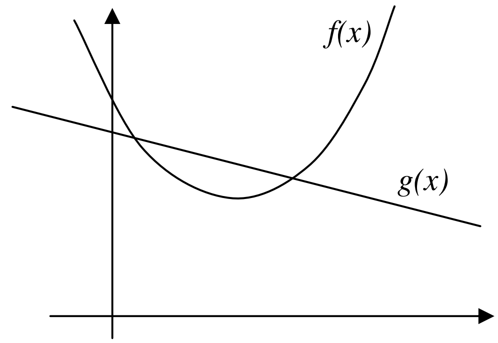
GOOD LUCK!

Do you want me to post your grade so far on the class website under the last 4 digits of your Student Number?

☐ *Yes, please post my grade. Sign to give permission:* ________________________________

☐ *No, please don't post my grade so far.*

Solutions Version 1



1 (15 points) The graphs of two functions, $f(x)$ and $g(x)$, are sketched on the right. The graph of $f(x)$ is a parabola with formula:

$$f(x) = x^2 - 6x + 13.$$

The graph of $g(x)$ is a straight line with y-intercept 10 and which passes through the point of coordinates (4,5).

a) (4 pts) Find the linear formula for $g(x)$, in terms of x.

Work:

g(x) goes thru the points (0, 10) and (4,5)
Slope=(5-10)/(4-0)= -5/4= -1.25

Answer: $g(x)$= -1.25 x + 10

b) (4 pts) Compute the values of x at which the two graphs cross.

Work:

Set f(x)=g(x) and solve for x:
x^2 - 6x +13= -1.25 x+10
x^2 -4.75x +3=0
Use the quadratic formula: x = 0.75 & x =4.

Answer: The two graphs cross at x =__0.75__and at x =__4__.

c) (3 pts) Find the longest interval, starting at x = 0, over which f(x) is decreasing.

Work:

Being concave-up, f(x) decreases up to its vertex. Use the vertex formula to get: $x = -\dfrac{-6}{2\times 1} = 3$

Answer: f(x) decreases from x=0 to x=_3__.

d) (4 pts) Find a formula (in terms of x) for $f(x+1) - f(x)$. Simplify it as much as possible.

Work:

$$f(x+1) = (x+1)^2 - 6(x+1) + 13 = x^2 + 2x + 1 - 6x - 6 + 13 = x^2 - 4x + 8$$
$$f(x+1) - f(x) = [x^2 - 4x + 8] - [x^2 - 6x + 13] = x^2 - 4x + 8 - x^2 + 6x - 13 = 2x - 5$$

Answer: $f(x+1) - f(x)$=_____2x-5_______

Solutions Version 1

2 (14 points) The following function gives the distance (in miles) traveled by bicyclist Bob after t hours, during a three hour trip.

$$\text{Distance: } B(t) = t^3 - 8t^2 + 25t$$

a) (3 pts) Find a formula (in terms of *t*) for Bob's average trip speed at time *t* hours.

Work:

$$ATS = \frac{B(t)}{t} = \frac{t^3 - 8t^2 + 25t}{t} = t^2 - 8t + 25$$

Answer: ATS(*t*) = ___t^2-8t+25________ mph

b) (4 pts) What was Bob's lowest average trip speed (ATS) during this trip?

Work:

Since Bob's ATS is a concave-up parabola, it decreases until its vertex.
Use the vertex formula to get the x-coordinate of the vertex: t=-(-8)/2=4 hrs.

In this version of the exam, the correct solution would've been to say that ATS is a concave-up parabola, so it decreases up to its vertex at t=4 hrs, but, since the trip is only 3 hours long, the lowest ATS for the duration of this trip will be at t=3 hrs, i.e. ATS(3)=$(3)^2$-8(3)+25= 10mph.

However, we also accepted answers that computed the vertex: ATS(4)= 4^2-8(4)+25= 9 mph.

Answer: Bob's lowest ATS was ______10___mph

c) (4 pts) Another biker, Anne, starts at the same place as Bob, and rides at a **linear instantaneous speed** given by the formula $s(t) = -1.6t + 20$ (in mph, with t in hrs). How far does Anne ride in the first half hour?

Work:

The distance Anne rides up to time t is A(t)= - 0.8t^2+20t (see WS 16).
A(0.5)=-0.8$(0.5)^2$+20(0.5)=9.8

Answer: Anne rides a distance of _9.8___ miles in the first half hour.

d) (3 pts) Set up (but DO NOT SOLVE) the equation you would need to solve in order to find the time when Anne is 2 miles ahead of Bob. Put your equation in the form at^3+bt^2+ct+d=0, as indicated in the answer.

Work:

Anne is 2 miles ahead of Bob when: A(t)=B(t)+2
- 0.8t^2+20t = t^3-8t^2+25t+2
Moving all on one side or the other of the equality:
t^3-7.2t^2+5t+2=0 OR -t^3+7.2t^2-5t-2=0 (either are OK)

Answer: (1)t^3+(-7.2)t^2+(5)t+(2)=0

Solutions Version 1

[3] (15 points) You are producing and selling bottles of Zen Yuppie Drink, in quantities from 1 to 500. If you sell q bottles, your total revenue (in dollars) is given by the function:

$$TR(q) = -0.0015q^2 + 3.5q.$$

Your total cost for producing q bottles (also in dollars) is:

$$TC(q) = 0.007q^2 + 0.5q + 125.$$

a) (3 pts) What is your fixed cost?

Work: TC(0)=125

Answer: ____125___dollars

b) (6 pts) Find all the bottle quantities q at which your average cost AC(q) does not exceed \$2.50 per bottle.

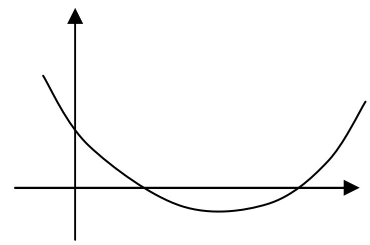
Work: $AC(q) = \frac{TC(q)}{q} = \frac{0.007q^2 + 0.5q + 125}{q}$.

$$\frac{0.007q^2 + 0.5q + 125}{q} \le 2.50$$

Multiply both sides by q: $0.007q^2 + 0.5q + 125 \le 2.50q$

Subtract 2.5q: $0.007q^2 - 2q + 125 \le 0$

This is a concave-up parabola (see sketch on right).
It's less than zero between its roots.
We can find its roots via the quadratic formula: q=92.35 & q=193.36
So AC≤2.50 for q between 93 and 193 bottles



Answer: ____ from 93 to 193____bottles.
(your answer should be a range of whole numbers of bottles)

c) (6 pts) How many bottles do you have to sell in order to maximize your profit? (*note: your answer should be a whole number of bottles*). What is your maximum profit?

Work:

Profit(q)=TR(q)-TC(q)= $-0.0015q^2 + 3.5q - (0.007q^2 + 0.5q + 125) = -0.0085q^2 + 3q - 125$

This is a concave-down parabola, so its max value is at its vertex:

$$q = -\frac{3}{2(-0.0085)} = \frac{3}{0.017} \cong 176.47$$

The whole number bottles at which the profit is largest is that nearest to 176.47, that is at q=176

Profit(176)=-0.0085(176)^2+3(176)-125=139.7

Answer: The profit is maximal at ___176___ bottles. The largest possible profit is __139.7___dollars.

Solutions Version 1

4 (6 points)

You are the owner of a company which produces and sells Cookies.
The selling price is always $\$p$ per Cookie, regardless of the quantity sold.
It costs you $\$c$ to make each Cookie, and your fixed cost is $\$f$.
(p, c and f are parameters – they stand in for specific numbers)

a) (2 pts) Write the formulas for your total revenue and your total cost at quantity q.
Your formulas should be in terms of the variable q, and the parameters p, f, and c. No explanation is needed.

TR(q)=__pq__________________

TC(q)= __cq+f__________________

b) (4 pts) Find the formula for the quantity q at which the total revenue is twice the total cost: TR(q)=2TC(q).
Your formula should be in terms of the parameters p, c and f.

Work:

TR(q)=2TC(q)
pq=2(cq+f)
pq=2cq+2f
pq-2cq=2f
(p-2c)q=2f
q=2f/(p-2c)

(assuming p>2c, otherwise there is no such quantity)

Answer: $q = \frac{2f}{p-2c}$.