VERSION 1

1. $TC(q) = \underbrace{q^2+5q}_{\text{variable cost}} + \underbrace{10}_{\text{Fixed cost}}$, $TR(q) = 25q$

(a) $TC(q) = VC(q) + FC$ so $VC(q) = TC(q) - FC$

Note: $FC = TC(0) = 10$

So $VC(q) = q^2 + 5q$

Thus, $VC(4) = (4)^2 + 5(4) = 16 + 20 =$ **36 dollars**

(b) $MC(q) = TC(q+1) - TC(q)$

$= [(q+1)^2 + 5(q+1) + 10] - [q^2 + 5q + 10]$

$= [q^2 + 2q + 1 + 5q + 5 + 10] - q^2 - 5q - 10$

$= \boxed{2q + 6}$

(c) **TWO METHODS**

Method 1: Find vertex of the profit function.

$P(q) = TR(q) - TC(q)$

$= [25q] - [q^2 + 5q + 10]$

$= 25q - q^2 - 5q - 10$

$P(q) = -q^2 + 20q - 10$

VERTEX $q = -\frac{b}{2a} = -\frac{20}{2(-1)} =$ **10 items**

$P(10) = -(10)^2 + 20(10) - 10$

$= -100 + 200 - 10 =$ **\$90**

Method 2: Solve $MR(q) = MC(q)$

$MR(q) = TR(q+1) - TR(q)$

$= 25(q+1) - 25q = 25q + 25 - 25q = 25$

$MR = MC \Rightarrow 25 = 2q + 6$

$19 = 2q \Rightarrow q = 9.5$

ROUND UP! $\Rightarrow$ **$q = 10$ items**

GET PROFIT SAME AS ABOVE **$P(10) =$ \$90**

(d) $TR(q) - TC(q) = 26$

$-q^2 + 20q - 10 = 26$

$-q^2 + 20q - 36 = 0$

$-(q-2)(q-18) = 0$

QUADRATIC FORMULA OR FACTOR **$q = 2, 18$**

VI

2 $D_A(t) = t^3 - 7t^2 + 20t$

$ATS_B(t) = 9 + 2t$

(a) $ATS_A(t) = \frac{D_A(t)}{t} = \frac{t^3 - 7t^2 + 20t}{t} = \boxed{t^2 - 7t + 20}$

$D_B(t) = t\ ATS_B(t) = \boxed{9t + 2t^2}$

(b) VERTEX OF $ATS_A(t)$

$t = -\frac{b}{2a} = -\frac{-7}{2(1)} = \boxed{3.5 \text{ hours}}$

(c) Find $\frac{D_B(5) - D_B(2)}{5 - 2}$

$D_B(5) = 9(5) + 2(5)^2 = 95$

$D_B(2) = 9(2) + 2(2)^2 = 26$

$\frac{D_B(5) - D_B(2)}{5 - 2} = \frac{95 - 26}{3} = \frac{69}{3} = \boxed{23 \text{ mph}}$

(d) Solve $ATS_A(t) = ATS_B(t) + 5$

$t^2 - 7t + 20 = 9 + 2t + 5$

$t^2 - 7t + 20 = 14 + 2t$

$t^2 - 9t + 6 = 0$

QUADRATIC FORMULA $t = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(6)}}{2(1)}$

$= \frac{9 \pm \sqrt{81 - 24}}{2}$

$= 8.27,\ 0.725$

$t = \boxed{8.27}, \boxed{0.73}$ hours.

VI

3 $p = 15 - 0.2q$, $AC(q) = 3 + \frac{20}{q}$

(a) $TR(q) = q(15 - 0.2q) = \boxed{15q - 0.2q^2}$

$TC(q) = q(3 + \frac{20}{q}) = \boxed{3q + 20}$

(b) $AVC(q) = \frac{VC(q)}{q} = \frac{3q}{q} = \boxed{3}$

Note: $VC(q) = 3q$ and $FC = 20$.

(c) Find the vertex of TR and compute the value

$q = -\frac{b}{2a} = -\frac{15}{2(-0.2)} = 37.5$ hundred things

$TR(37.5) = \boxed{281.25}$ hundred dollars

(d) $MC(5) = TC(5.01) - TC(5)$

$= [3(5.01) + 20] - [3(5) + 20]$

$= 35.03 - 35$

$= 0.03$ hundred dollars

$\boxed{\$3.00}$

(e) Solve $AC(q) = 14$

$3 + \frac{20}{q} = 14$

$\frac{20}{q} = 11$

$20 = 11q$

$q = \frac{20}{11} = 1.8181\ldots$

$\boxed{182 \text{ Things}}$

ASIDE

As stated in the problem, q is in hundreds of Things. So in (d): 500 to 501 is $q = 5$ to $q = 5.01$.

in (e): $q = 1.818$ is 182 Things.