V1

Math 111 - Winter 2009 - Exam 2 - Version 1

1 (a) $ATS(t) = \frac{D(t)}{t}$

$ATS_A(t) = \frac{D_A(t)}{t} = \frac{t^3 - 7t^2 + 20t}{t}$

$\boxed{ATS_A(t) = t^2 - 7t + 20}$

$ATS_B(t) = \frac{D_B(t)}{t} = \frac{70t - 2t^2}{t}$

$\boxed{ATS_B(t) = 70 - 2t}$

(b) $D_B(t) = 50 \Rightarrow 70t - 2t^2 = 50 \Rightarrow -2t^2 + 70t - 50 = 0$

By the quadratic formula, $t = \frac{-70 \pm \sqrt{70^2 - 4(-2)(-50)}}{2(-2)} = \frac{(-70 \pm \sqrt{4500})}{-4}$

$\underbrace{t = 0.7294901688\text{ hrs}}_{\text{first time}}$ or $t = 34.27050983$ hrs.

$\boxed{t = 0.73 \text{ hrs}}$

(c) $ATS_B(t) = 25 \Rightarrow 70 - 2t = 25 \Rightarrow -2t = -45$

$\Rightarrow \boxed{t = \frac{-45}{-2} = 22.5 \text{ hrs}}$

(d) Smallest value of $ATS_A(t) = t^2 - 7t + 20$ occurs at the vertex $t = -\frac{b}{2(a)} = -\frac{-7}{2(1)} = 3.5$ hrs.

$\boxed{ATS_A(3.5) = 7.75 \text{ mph}}$

Smallest value of $ATS_B(t) = 70 - 2t$ occurs at $t = 20$ because $ATS_B(t)$ is a line with a negative slope. (so it gets smaller going to the right).

$\boxed{ATS_B(20) = 70 - 2(20) = 30 \text{ mph}}$

2 $\frac{D_B(t+2) - D_B(t)}{2} = \frac{[70(t+2) - 2(t+2)^2] - [70t - 2t^2]}{2}$

$= \frac{\cancel{70t} + 140 - 2(t^2 + 4t + 4) - \cancel{70t} + 2t^2}{2}$

$= \frac{140 - 8t - 8}{2} = \frac{132 - 8t}{2} = \boxed{66 - 4t}$

3 (a) Since $AVC(q) = \frac{VC(q)}{q}$, we know $VC(q) = q\,AVC(q)$.

Hence, $\boxed{VC(q) = q\left(\frac{1}{3}q^2 - 5q + 19\right) = \frac{1}{3}q^3 - 5q^2 + 19q}$

Since $TC(q) = VC(q) + FC$ and $FC = 2$ hundred dollars,

$\boxed{TC(q) = \frac{1}{3}q^3 - 5q^2 + 19q + 2}$

Since $AC(q) = \frac{TC(q)}{q}$, we have

$\boxed{AC(q) = \frac{1}{3}q^2 - 5q + 19 + \frac{2}{q}}$

(b) Solve $MR = MC$ and round up to a whole # of items.

$25 - 4q = q^2 - 10q + 19 \Rightarrow 0 = q^2 - 6q - 6$

The quadratic formula gives $q = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(-6)}}{2}$

$q = \frac{6 \pm \sqrt{60}}{2} = 3 \pm \sqrt{15}$

$q = -0.872983346$ or $q = 6.872983346$

At $q = 6.872983346$ there is a transition from $MR > MC$ to $MR < MC$, so the max profit occurs at $q = 6.872983346$ which rounds up to $\boxed{688 \text{ Items}}$

(c) SDP = value of AVC when it's at its lowest

= value of AVC when MC = AVC.

method 1 Find vertex of AVC

AVC is a quadratic. ✓

Vertex: $q = -\frac{b}{(2a)} = -\frac{-5}{2(\frac{1}{3})} = 7.5$

$AVC(7.5) = \frac{1}{3}(7.5)^2 - 5(7.5) + 19$

$= 0.25$

method 2 Solve MC = AVC.

$q^2 - 10q + 19 = \frac{1}{3}q^2 - 5q + 19$

$\frac{2}{3}q^2 - 5q = 0$

$q\left(\frac{2}{3}q - 5\right) = 0$

$q = 0$ or $\frac{2}{3}q - 5 = 0$

$q = 7.5$

$AVC(7.5) = 0.25$

$\boxed{SDP = 0.25 \text{ dollars per Item}}$

4 (a) $\boxed{TR(q) = pq = (22-2q)q = 22q - 2q^2}$

$TC(q) = mq + b \qquad (0,3) \; ; \; (4,19)$

so $m = \frac{19-3}{4-0} = \frac{16}{4} = 4 \qquad b = 3$

$\boxed{TC(q) = 4q + 3}$

(b) $MR(4) = TR(4.01) - TR(4)$

$= [22(4.01) - 2(4.01)^2] - [22(4) - 2(4)^2]$

$= 56.0598 - 56 = 0.0598$ hundreds of dollars

$= \boxed{5.98 \text{ dollars}}$

(c) $TR(q) = TC(q)$

$22q - 2q^2 = 4q + 3 \quad \Rightarrow \quad -2q^2 + 18q - 3 = 0$

The quadratic formula gives

$q = \frac{-18 \pm \sqrt{18^2 - 4(-2)(-3)}}{2(-2)} = \frac{-18 \pm \sqrt{300}}{-4}$

$= 0.1698729811 \quad \text{or} \quad 8.830127019$

$\boxed{17 \text{ shirts and } 883 \text{ shirts}}$

(d) $P(q) = TR(q) - TC(q)$

$= [22q - 2q^2] - [4q + 3]$

$= -2q^2 + 22q - 4q - 3 = -2q^2 + 18q - 3$

Profit is a quadratic that opens downward so the vertex is at the maximum

$q = -\frac{b}{(2a)} = -\frac{18}{2(-2)} = 4.5 \qquad (450 \text{ shirts})$

$P(4.5) = -2(4.5)^2 + 18(4.5) - 3 = 37.50$

$\boxed{\begin{array}{l} 450 \text{ shirts} \\ 3750.^{\underline{00}} \text{ dollars} \end{array}}$