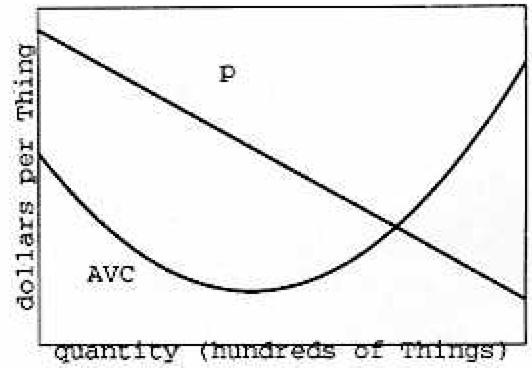
1. (16 points)

You sell Things. The price, p, in dollars per Thing and the average variable cost, AVC, in dollars per Thing on an order of q **hundred** Things are

$$p(q) = 28 - 3q \quad \text{and} \quad AVC(q) = q^2 - 9q + 22.$$

The **fixed cost** is \$1,800. ($FC = 18$ hundred dollars). The graphs of price, p, and average variable cost, AVC, are given.



(a) (4 pts) Write out the formulas for the total revenue, variable cost, total cost, and average cost for selling q hundred Things.

$TR(q) = pq$

$VC(q) = q\,AVC(q)$

$TC(q) = VC(q) + FC$

$AC(q) = \frac{TC(q)}{q}$

$TR(q) = (28-3q)q = \boxed{28q - 3q^2}$

$VC(q) = q(q^2-9q+22) = \boxed{q^3 - 9q^2 + 22q}$

$TC(q) = \boxed{q^3 - 9q^2 + 22q + 18}$

$AC(q) = \boxed{q^2 - 9q + 22 + \frac{18}{q}}$

(b) (4 pts) Find the profit for selling 460 Things.
(Give your final answer in dollars rounded to the nearest cent.)

$q = 4.6$ hundred Things

PROFIT $= TR(4.6) - TC(4.6)$

$= [28(4.6) - 3(4.6)^2] - [(4.6)^3 - 9(4.6)^2 + 22(4.6) + 18]$

$= 65.32 - 26.096$

$= 39.224$ hundred dollars

$\boxed{3922.40}$ dollars

(c) (4 pts) Find the largest interval of quantities over which the average variable cost is less than or equal to 6 dollars per Thing. (Your values of q should be accurate to the nearest Thing).

$q^2 - 9q + 22 = 6 \Rightarrow q^2 - 9q + 16 = 0 \qquad a=1, b=-9, c=16$

$$q = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(16)}}{2(1)} = \frac{9 \pm \sqrt{17}}{2} \approx 2.43844 \text{ or } 6.56155$$

$q = \boxed{2.44}$ **hundred Things** to $q = \boxed{6.56}$ **hundred Things**

(d) (4 pts) Find the Shutdown Price (SDP).

SDP = "lowest value of AVC"

$AVC(q) = q^2 - 9q + 22$

VERTEX: $q = -\frac{-9}{2(1)} = 4.5$

$AVC(4.5) = (4.5)^2 - 9(4.5) + 22$

$= 1.75$

$\boxed{1.75}$ dollars per item

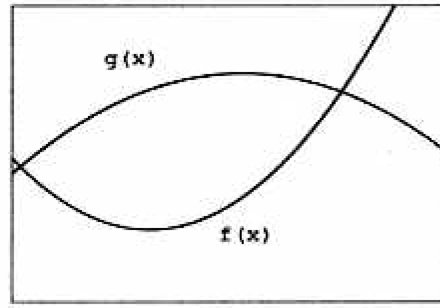
V1

2. (19 points)

The graphs to the right are parabolas with formulas:

$$f(x) = x^2 - 6x + 18, \quad \text{and}$$

$$g(x) = -\tfrac{1}{2}x^2 + 5x + 16.$$



Give all final answer accurate to two digits after the decimal.

(a) (4 pts) Find all values of x at which the two graphs cross.

$x^2 - 6x + 18 = -\frac{1}{2}x^2 + 5x + 16$

$1.5x^2 - 11x + 2 = 0$

$x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(1.5)(2)}}{2(1.5)}$

$= \frac{11 \pm \sqrt{109}}{3} = 0.18656$ or 7.14676

$\boxed{x = 0.19 \text{ or } 7.15}$

(b) (4 pts) Find the longest interval when $f(x)$ and $g(x)$ are both increasing.

VERTEX OF $f(x)$: $x = -\frac{-6}{2(1)} = 3$ $\quad$ $f(x)$ increases after $x = 3$

VERTEX OF $g(x)$: $x = -\frac{5}{2(-1/2)} = 5$ $\quad$ $g(x)$ increases before $x = 5$

$x =$ $\boxed{3}$ to $x =$ $\boxed{5}$

(c) (4 pts) Find the size of the biggest vertical gap when $g(x)$ is above $f(x)$. (That is, find the largest value of $g(x) - f(x)$.)

$g(x) - f(x) = [-\frac{1}{2}x^2 + 5x + 16] - [x^2 - 6x + 18]$

$= -1.5x^2 + 11x - 2$

VERTEX: $x = -\frac{11}{2(-1.5)} = \frac{11}{3} = 3.\overline{6}$

$y = g(3.\overline{6}) - f(3.\overline{6}) = 18.1\overline{6}$

$\boxed{18.17}$

(d) Let $h(x)$ be a new parabola given by the relationship $h(x) = f(x-4)$.

i. (4 pts) Write out the formula for $h(x) = f(x-4)$ and simplify into the expanded quadratic form, $h(x) = (\ \ )x^2 + (\ \ )x + (\ \ )$.

$f(x-4) = (x-4)^2 - 6(x-4) + 18$

$= x^2 - 8x + 16 - 6x + 24 + 18$

$= x^2 - 14x + 58$

$h(x) =$ $\boxed{x^2 - 14x + 58}$

ii. (3 pts) Find the x and y values of the vertex for $h(x)$. (Hint: There is a way to answer using the previous part and a way to answer directly from $f(x)$. Either way is fine, just show and explain your work.)

$x = -\frac{-14}{2(1)} = 7$

$y = (7)^2 - 14(7) + 58 = 9$

$x =$ $\boxed{7}$

$y =$ $\boxed{9}$

SAME AS VERTEX OF $f(x)$ BUT SHIFTED RIGHT 4

VI

3. (15 pts) The total revenue for Items is given by $TR(q) = -4q^2 + 78q$, where q is in Items and $TR(q)$ is in dollars.
The fixed cost (FC) is 150 dollars and the marginal cost (MC) is always a constant 8 dollars per item. (In other words, $TC(0) = 150$, $TC(1) = 158$, $TC(2) = 166$, *etc.*)

(a) (2 pts) From the description, total cost (TC) is a linear function (*i.e.* $TC(q) = mq + b$). Give the formula for Total Cost.

$TC(q) =$ $\boxed{8q + 150}$

(b) (4 pts) Find the largest quantity when the price per item is equal to the average cost per item. And give the value of profit at this item.

MEANS PROFIT = 0

price = AC

$\frac{TR(q)}{q} = \frac{TC(q)}{q}$ (SAME AS: $TR(q) = TC(q)$)

$-4q + 78 = 8 + \frac{150}{q}$

$-4q^2 + 78q = 8q + 150$

$-4q^2 + 70q - 150 = 0$

$q = \frac{-70 \pm \sqrt{70^2 - 4(-4)(-150)}}{2(-4)} = \frac{-70 \pm 50}{-8}$
$= 2.5$ or 15

$q =$ $\boxed{15}$ items

Profit at these quantities = $\boxed{0}$ dollars

(c) (5 pts) Recall $MR(q) = TR(q+1) - TR(q)$. Find and simplify the formula for marginal revenue.

$MR(q) = [-4(q+1)^2 + 78(q+1)] - [-4q^2 + 78q]$
$= \cancel{-4q^2} - 8q - 4 + \cancel{78q} + 78 \cancel{+4q^2} \cancel{-78q}$
$= -8q + 74$

$MR(q) =$ $-8q + 74$

(d) (4 pts) At what quantity q is the profit greatest? What is the greatest profit?

$\boxed{\text{METHOD 1}}$ MR = MC

$-8q + 74 = 8$

$-8q = -66$

$q = \frac{-66}{-8} = 8.25$

ROUND UP! $\boxed{q = 9}$

PROFIT = TR(9) − TC(9)
= 378 − 222 = $\boxed{156}$

$\boxed{\text{METHOD 2}}$ PROFIT = TR − TC
$= [-4q^2 + 78q] - [8q + 150]$
$= -4q^2 + 70q - 150$

VERTEX: $q = -\frac{70}{2(-4)} = 8.75$

$\boxed{q = 9}$

$q =$ $\boxed{9}$ items

Max Profit = $\boxed{156}$ dollars