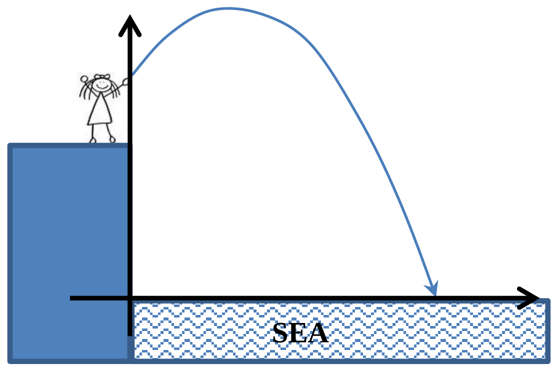
1. (13 pts) Mary stands on a cliff and throws a rock toward the sea. At t seconds from the moment the rock was thrown, its height $H(t)$ above the sea level is given by the formula:

$$H(t) = -16t^2 + 24t + 15 \text{ (in feet)}$$



a) (3 pts) How high above the sea level is the rock at the moment when Mary throws it?

$$H(0) = 15$$

ANSWER: 15 feet

b) (5 pts) What is the greatest height of the rock above the sea level?

The graph of $H(t)$ is a concave-down parabola, so greatest height occurs at its vertex.
Vertex Formula: $t = \frac{-24}{2(-16)} = 0.75\ seconds$
Greatest height is $H(0.75) = -16(.75)^2 + 24(.75) + 15 = 24$ feet

ANSWER: 24 feet

c) (5 pts) Compute the longest time interval during which the rock is at a height of at least 15 feet above the sea level.

$$H(t) = 15 \text{ feet}$$
$$-16t^2 + 24\,t + 15 = 15$$
$$-16t^2 + 24\,t = 0$$

Using the Quadratic Formula (or factoring), we get: $t = 0$ or $t = 1.5$

So the height is 15 feet at t=0 and t=1.5 seconds.

Since its graph is concave-down, the height will be above 15 feet between the two solutions.

ANSWER: from _0_ to _1.5_ seconds

2. (13 pts) The **average trip speed** (ATS) of a certain car at *t* seconds is given by the following function:

$$s(t) = \frac{10}{t} + 60 \text{ feet/sec.}$$

a) (3 pts) Find the formula, in terms of the time t, for the distance function $D(t)$ of this car.

$$D(t) = t \times s(t) = t \times \left(\frac{10}{t} + 60\right) = 10 + 60t$$

ANSWER: $D(t) = 60t + 10$

b) (4 pts) Compute the average speed of this car over the time interval from 1 to 2.5 seconds.

Method 1: $AS = \frac{D(2.5)-D(1)}{2.5-1} = \frac{(60(2.5)+10)-(60(1)+10)}{1.5} = \frac{160-70}{1.5} = \frac{90\ feet}{1.5\ sec} = 60\ ft/sec$

Method 2: Since D(t) is a line, its rate of change is constant, so the average speed over any time interval is equal to the slope, i.e. to 60.

ANSWER: $AS = 60$ feet/sec

c) (6 pts) Another car has the following formula for its average trip speed *vs.* time: $p(t) = 1.5t + 2$ ft/sec. Find the time *t* when the two cars have **the same average trip speed**.

$$s(t) = p(t)$$

$$\frac{10}{t} + 60 = 1.5t + 2$$

$$\frac{10}{t} = 1.5t - 58$$

$$t \times \frac{10}{t} = t \times (1.5t - 58)$$

$$10 = 1.5t^2 - 58t$$

$$1.5t^2 - 58t - 10 = 0$$

Quadratic Formula: $t = -0.17165 \ldots$ or $t = 38.8383 \ldots$

Answer: At *t* =___38.84__ seconds.

3. (12 pts) You produce and sell Widgets. Each Widget costs \$7 to produce.

Your Marginal Revenue is a linear function, with $MR(1) = \$8$, and $MR(10) = \$7.19$.

a) Compute the linear formula for the Marginal Revenue function, in terms of the quantity q.

Find the equation of the line through the points (1, 8) and (10, 7.19):

$$Slope = \frac{7.19 - 8}{10 - 1} = -\frac{0.81}{9} = -0.09$$

So: $MR(q) = -0.09q + b$. Use one of the points to compute the y-intercept:

$MR(1) = 8 = -0.09(1) + b \ => b = 8.09$.

ANSWER: $MR(q) = -0.09q + 8.09$

b) What number of Widgets produced and sold results in the maximum profit?

Your answer should be a whole number.

$$MR(q) = MC(q)$$

$$-0.09q + 8.09 = 7$$

$$-0.09q = -1.09$$

$$q = 12.111 \ldots$$

Round up to the next whole number of Widgets

ANSWER: $q = 13$ Widgets

4. (12 pts) You produce and sell Trinkets. Your total revenue, in dollars, from selling q Trinkets is

$$TR(q) = -2q^2 + 18q.$$

a) Recall that $MR(q) = TR(q+1) - TR(q)$. Compute the formula for $MR(q)$ in terms of q. Simplify it as much as possible.

$$\begin{aligned} &TR(q+1) - TR(q) = \\ &= [-2(q+1)^2 + 18(q+1)] - [-2q^2 + 18q] \\ &= -2(q^2 + 2q + 1) + 18(q+1) + 2q^2 - 18q \\ &= -2q^2 - 4q - 2 + 18q + 18 + 2q^2 - 18q \\ &= -4q + 16 \end{aligned}$$

ANSWER: $MR(q) = -4q + 16$ $/Item

b) Suppose your variable cost, in dollars, is given by

$$VC(q) = q^3 - 10q^2 + 32q$$

Compute the Shutdown price.

$$SDP = \text{lowest value of } AVC$$

$$AVC(q) = \frac{VC(q)}{q} = \frac{q^3 - 10q^2 + 32q}{q} \Rightarrow \boxed{AVC(q) = q^2 - 10q + 32}$$

Concave-up parabola => lowest value is at $q = \frac{8}{2} = 5$ items.

$$SDP = AVC(5) = 5^2 - 10(5) + 32 = 7$$

ANSWER: $SDP = 7$ $/Item