MATH 111A – Autumn 2001

Final Exam

Hints and Answers

1. (a) HINT: Solve $980,000 = 460,000(1+r)^9$ for r and convert to a percentage.
   ANSWER: 8.77%

   (b) HINT: Let T be the tuition this year. Then, next year's tuition will be $T + 0.059T = 1.059T$. Similarly, the next year's tuition will be $1.059(1+r) \cdot T$. This pattern will continue, so that the tuition 5 years from now will be $1.059(1+r)^4 \cdot T$. You want that to be $T + 0.35T$ or $1.35T$. So, solve the equation

   $$1.35T = 1.059(1+r)^4 \cdot T$$

   for r.
   ANSWER: $r = 0.0625747$

   (c) HINT: Solve $3000 = P\left(1 + \dfrac{0.0225}{12}\right)^{12(10/12)}$ for P.
   ANSWER: $P = \$2944.33$

   (d) HINT: Solve $70 = Pe^{0.12(0.5)}$ for P and $100 = Pe^{0.12(1)}$ for P. Add the two amounts.
   ANSWER: $154.61

2. (a) HINT: Solve $17,500 = 10,000\left(1 + \dfrac{0.06015}{4}\right)^{4t}$ for t. First, divide both sides by 10,000 and take the natural log of both sides.
   ANSWER: 9.37 years

   (b) ANSWER: Account I: APY=6.15%; Account II: APY=6.18%

   (c) HINT: After 2 years, your balance in Account I has become $11,268.26. You then need to solve $17,500 = 11,268.26e^{0.06t}$ for t.
   ANSWER: We would accept either 7.34 years or a total of 9.34 years.

3. (a) ANSWER: $A_{10} = A_8 + 6$

   (b) ANSWER: $A_k = 57 + 3(k-1)$

   (c) HINT: $B_1 = 7^{57}$, $B_2 = 7^{57+3} = 7^{57} \cdot 7^3$, $B_3 = 7^{57+3+3} = 7^{57} \cdot 7^3 \cdot 7^3$, ...
   ANSWER: B is a multiplicative sequence with multiplier 7^3.

4. (a) HINT: Draw a diagonal line that is tangent to the graph of VC and compute its slope.
   ANSWER: We would accept answers between $0.40 and $0.50.

   (b) ANSWER: We would accept answers between 550 and 575. (Your answer should be the quantity at which your tangent line from part (a) touches the graph of VC.)
   EXPLANATION: The line you drew in part (a) is both a diagonal line and a tangent line. So, it's slope is both a value of AVC and a value of MC. The quantity at which these values are equal is the quantity at which this diagonal/tangent line intersects the VC graph.

   (c) HINT: The graph of TC should look like the graph of VC shifted up 300.

   (d) ANSWER: I choose graph (iii) because, if I use the rolling ruler method to investigate the slopes of diagonal lines to the VC graph (AVC=the slope of a diagonal line to VC), I can see that these slopes decrease for a while and then start to increase. Graph (iii) is the only choice that fits that pattern.

5. (a) HINT: The time 8 a.m. corresponds to $t = 2$ and the time 10 a.m. to the time $t = 4$. The graph shows that the average rate of change over this 2-hour period is 4° per hour. A change of 4° per hour for 2 hours is 8°.
   ANSWER: 8°

(b) HINT: From $t = 4$ to $t = 6$, the temperature changed at a rate of 2 degrees per hour. So, $P(6) - P(4) = 4$ degrees. Similarly, the temperature changed at a rate of 0.5 degrees per hour from $t = 6$ to $t = 8$. So, $P(8) - P(6) = 1$ degree. You are looking for $P(8) - P(4)$, the change in temperature from $t = 4$ to $t = 8$.

ANSWER: $P(8) - P(4) = 5$

(c) ANSWER: I recorded the highest temperature at $t = 8$ (or 2 p.m.).

EXPLANATION: The termperature increased over any two-hour period when the rate of change is positive. So, the temperature increased from $t = 0$ to $t = 2$, from $t = 2$ to $t = 4$, from $t = 4$ to $t = 6$, and from $t = 6$ to $t = 8$. From $t = 8$ to $t = 10$, the temperature began to decrease (and kept decreasing until midnight).

6. (a) HINT: Since the graph goes through the point $(0, 2)$, $f(0) = 2$. But $f(0) = a(0)^2 - (0) + c$. So, $c = 2$. That gives us $f(x) = ax^2 - x + 2$. Similarly, use the fact that $f(2) = 1$ to find the value of a.

ANSWER: $a = \frac{1}{4}$, $c = 2$

(b) HINT: You should draw a line that goes through the graph at $x = 2$ and $x = 2.5$. Use your answer to (a) to determine that $f(2.5) = 1.0625$. You know that $f(2) = 1$. Then, use the formula for the slope.

ANSWER: slope=0.125

(c) HINT: $f(3 + h) = \frac{1}{4}(3 + h)^2 - (3 + h) + 2$ Multiply out and combine like terms.

ANSWER: $f(3 + h) = \frac{1}{4}h^2 + \frac{1}{2}h + \frac{5}{4}$