MATH 111A – Autumn 2001

Final Exam

Hints and Answers

1. (a) ANSWER: 8.87%

   (b) HINT: Solve $1300 = Pe^{0.085(1.5)}$ for P.

   ANSWER: \$1144.38

   (c) HINT: Solve $1500 = 1200e^{0.085t}$ for t. Start by dividing both sides by 1200. Then take the natural log of both sides.

   ANSWER: $t = 2.63$ years

2. (a) HINT: Solve $63 = 2(1 + r)^{15}$ for r and convert to a percentage.

   ANSWER: 25.86%

   (b) HINT: After t years, your yearly rent will be $(1.0512)^t \cdot R$. You want to know when this is equal to $3R$. So, solve $3R = (1.0512)^t \cdot R$ for t. Start by dividing both sides by R and then take the natural log of both sides.

   ANSWER: 22 years

   (c) HINT: After 7 years in the first account, your balance is \$1139.84. Place that in the second account for 3 years.

   ANSWER: \$1284.91

3. (a) ANSWER: $A_{10} = 3^2 \cdot A_8$

   (b) ANSWER: $A_k = 12 \cdot 3^{k-1}$

   (c) HINT: $B_2 = \log_{10}(A_2) = \log_{10}(A_1 \cdot 3) = \log_{10}(A_1) + \log_{10}(3) = B_1 + \log_{10} 3$.

   ANSWER: B_k is additive with increment $\log_{10} 3$.

4. (a) HINT: $\frac{I(t)}{t} = \frac{1}{27}t^2 - \frac{1}{9}t + \frac{1}{3}$. Set this equal to $\frac{7}{3}$ and solve for t.

   ANSWER: $t = 9$

   (b) HINT: Use the formula for $I(t)$ to compute

   $$\frac{I(6.5) - I(1)}{6.5 - 1}.$$

   ANSWER: 1.34 gallons per hour

   (c) HINT: The amount in the tank is $A(t) = I(t) - O(t) + 15$.

   ANSWER: A(4)=6.93 gallons

   (d) HINT: You want $I(t) = 3 + O(t)$.

   ANSWER: We would accept any equation equivalent to

   $$\frac{1}{27}t^3 - \frac{1}{9}t^2 + \frac{1}{3}t = 3 + 2.5t.$$

5. (a) HINT: In the first week, Iggy grows at a rate of 0.3 inches per day.

   ANSWER: 2.1 inches

   (b) HINT: $L(7) - L(4)$ is the change in Iggy's length from the end of week 4 to the end of week 7. That is, you need to know how much Iggy grows during weeks 5, 6 and 7. During week 5, Iggy grows at a rate of 0.4 inches per day. So, he grows 0.4×7 inches that week. Do the same for weeks 6 and 7 and add up all the weekly changes.

   ANSWER: 9.1 inches

   (c) HINT: The graph of $L(t)$ should always be increasing (since Iggy is growing and, also, since the rates of change are all positive). So, that eliminates graphs (i) and (iv). To choose between (ii) and (iii), notice that, for (ii), the weekly rates of change would be very small during the first few weeks and then get larger. That is not the behavior of the incremental change graph that you are given.

   ANSWER: I choose (iii).

6. (a) HINT: Draw a diagonal line that is tangent to the TC graph and compute its slope.
   ANSWER: \$0.75 (approximately)

   (b) HINT: The graph of TR will be a diagonal line with slope 1.
   ANSWER: \$150 (approximately)

   (c) HINT: Draw a diagonal line that intersects the graph of TC at $q = 3$. This is the graph of TR. The price per item is the slope of the graph of TR.
   ANSWER: $p = \$1.80$ (approximately)