

MATH 111 A – Version 1
Final Exam
December 16, 2002

Name _____

Student ID # _____

Section _____

1	16	
2	20	
3	16	
4	23	
5	13	
6	12	
Total	100	

- You are allowed to use a calculator, a ruler, and one sheet of handwritten notes.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit.
- If you use a trial and error method when an algebraic method is available, you will not receive full credit.
- Write your answers in the specified locations.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so. If you still need more paper, please ask for some.
- Raise your hand if you have a question.
- You have 1 hour and 50 minutes to complete the exam.

GOOD LUCK!

1. (16 points) You are considering putting money into one of two accounts. The nominal period is one year.

Account A: nominal rate 3%, compounded annually

Account B: nominal rate 6%, compounded monthly

In each of the following questions, round your **final answer** to two digits after the decimal.

- (a) (4 points) You deposit \$100 in account *B*. What is the percentage change in your balance after 1 year?

ANSWER: _____%

- (b) (4 points) Compute the present value of \$1400 to be received from Account *A* 11 years from now.

ANSWER: \$_____

- (c) (4 points) Compute the length of time it would take to double your principal in Account *B*.

ANSWER: _____ years

- (d) (4 points) Suppose Account *B* is not available to you now, but it will be in 5 years. You deposit \$3000 in Account *A* for 5 years and then deposit the resulting balance in Account *B*. What is your balance after a **total** of 8 years?

ANSWER: \$_____

2. (20 points) In each of the following questions, round your **final answer** to two digits after the decimal.

(a) (5 points) A bottle-cap collector paid \$.55 for a Fanta Red Cream Soda cap in 1991. In 2002, the collector sells the cap for \$1.70. What is the annual rate of return for this investment?

ANSWER: _____%

(b) (5 points) Marsha loans her daughter Julie \$450 to buy a new amplifier. Julie pays the loan back in 21 months with interest of 3% per year (compounded annually). What is the total sum that Julie pays her mother?

ANSWER: \$ _____

- (c) (5 points) On the first Sunday of every month, Matt enjoys a bowl of oatmeal and a cinnamon roll at Lydia's Cafe. With tax and tip, his bill comes to \$13. He would like to endow his breakfast. That is, he will deposit some amount of money in an account and withdraw the interest each month and use it to pay for breakfast. How much must Matt deposit into an account paying 1.75% annual interest, compounded continuously, to earn \$13 *in interest* after one month?

ANSWER: \$_____

- (d) (5 points) The owner of a restaurant buys a new refrigerator for \$4300. Once it's put into use, the value of the refrigerator begins to depreciate. Each year the value of the refrigerator is 75% of its value the year before. After how many years is the value of the refrigerator \$2000?

ANSWER: _____ years

3. (16 points)

(a) (4 points) Describe how you determine whether or not a sequence is additive.

(b) (4 points) Suppose $A(k)$ is an additive sequence with increment 5. Give an equation that relates $A(k + 1)$ to $A(k)$.

ANSWER: _____

(c) (4 points) Let $A(k)$ be the sequence described in part (b). Let $B(k)$ be a sequence with the following property:

$$B(1) = 2A(1) + 4, B(2) = 2A(2) + 4, B(3) = 2A(3) + 4, \dots$$

$B(k)$ is also an additive sequence. Find its increment.

ANSWER: _____

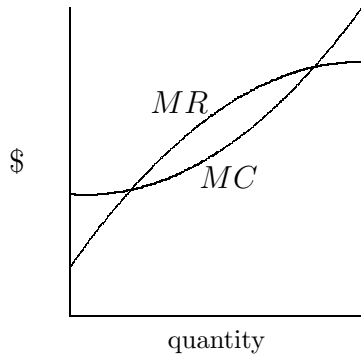
(d) (4 points) Let $B(k)$ be the sequence described in part (c). Suppose $B(1) = 22$. Compute the value of $B(108)$.

ANSWER: $B(108) =$ _____

4. (23 points) You sell items. The Marginal Revenue and Marginal Cost are given by the formulas:

$$MR(q) = -0.03q^2 + 1.42q + 4 \text{ and } MC(q) = 0.03q^2 - 0.08q + 10.$$

The quantity is in items and the MR and MC are in dollars. The graphs of MR and MC are given below.



- (a) (6 points) Find the two quantities at which $MR = MC$.

ANSWER: $q =$ _____ and $q =$ _____

- (b) (5 points) Profit is maximized at one of the quantities you found in part (a) but not at the other. Decide which quantity maximizes profit and give a reason for your choice.

ANSWER: Profit is maximized when $q =$ _____ items because:

We are still considering the functions

$$MR(q) = -0.03q^2 + 1.42q + 4 \text{ and } MC(q) = 0.03q^2 - 0.08q + 10.$$

(c) (4 points) If $MR = aq^2 + bq + c$, then the formula for Total Revenue is

$$TR = \frac{a}{3}q^3 + \frac{b}{2}q^2 + cq.$$

If $MC = aq^2 + bq + c$, then the formula for Variable Cost is

$$VC = \frac{a}{3}q^3 + \frac{b}{2}q^2 + cq.$$

Give the formulas for Total Revenue and Variable Cost for your items.

ANSWER: $TR(q) =$ _____

$VC(q) =$ _____

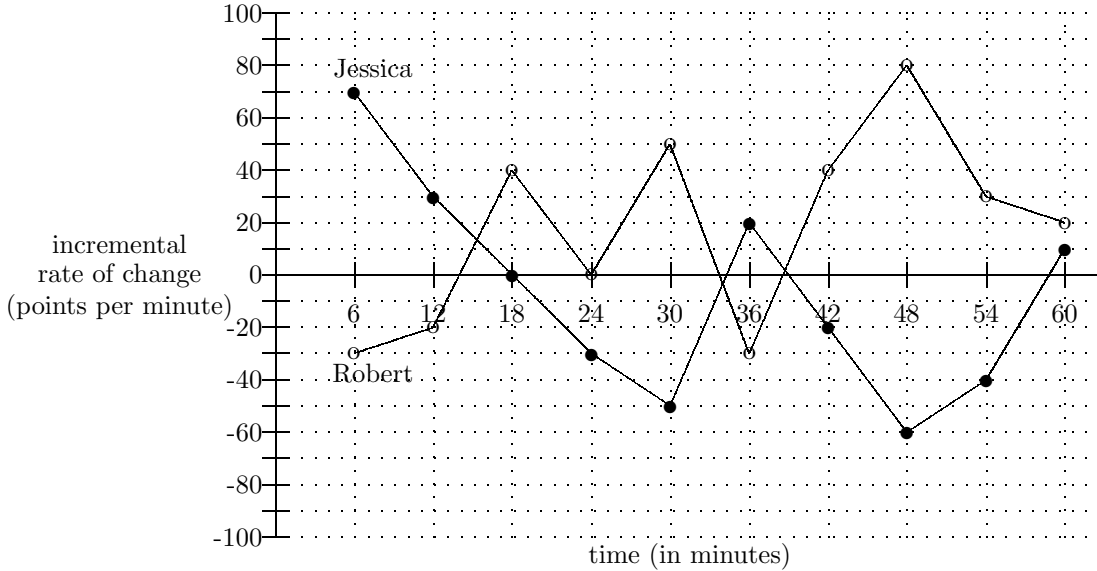
(d) (5 points) Which value of Fixed Cost will make the Total Cost of producing 6 items \$100?

ANSWER: $FC = \$$ _____

(e) (3 points) Set up an equation that you would solve in order to find the quantity at which Variable Cost is \$120 less than Total Revenue. Do not attempt to solve the equation.

ANSWER: _____

5. (13 points) Jessica and Robert are playing a game in which they can earn and lose points. At the start of the game, each player has 1000 points. Pat is watching the game. Every 6 minutes, Pat computes and plots the *incremental rate of change* in the number of each player's points over the *past* six minutes. Pat's graphs are given below. (For example, if $J(t)$ is Jessica's score after t minutes, then $\frac{J(6)-J(0)}{6} = 70$.)



- (a) (5 points) Which player is ahead after 18 minutes? By how much?

ANSWER: (circle one) [Jessica Robert] is ahead by _____ points

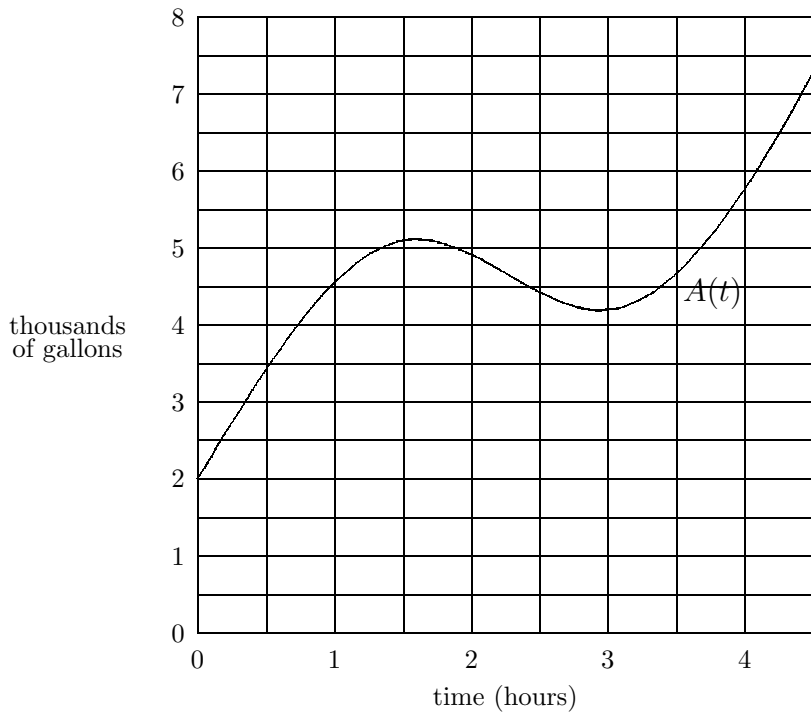
- (b) (4 points) During which 6-minute interval did Robert's score increase by 300 points?

ANSWER: from $t =$ _____ to $t =$ _____ minutes

- (c) (4 points) During which 6-minute interval does Jessica's score first fall below 1000 points?

ANSWER: between $t =$ _____ and $t =$ _____ minutes

6. (12 points) Below is the graph of the amount of water in a tank at time t . We denote the amount of water in the tank by $A(t)$.



- (a) (4 points) Find all times at which there are 4,500 gallons in the tank.

ANSWER: $t =$ _____

- (b) (4 points) The overall average rate of change of the amount of water in the tank is given by

$$R(t) = \frac{A(t) - A(0)}{t}.$$

Compute the overall average rate at $t = 3$. Round your answer to two digits after the decimal, if necessary.

ANSWER: $R(3) =$ _____ thousand gallons per hour

- (c) (4 points) Estimate the *incremental* rate of change in the amount of water over the time interval $t = 1$ to $t = 1.01$ hours. Round your answer to two digits after the decimal, if necessary.

ANSWER: _____ thousand gallons per hour