MATH 111A – Autumn 2002
Final Exam
Hints and Answers

1. (a) HINT: You earn \$6.17 in interest in one year. What percentage of your original deposit is this?
   ANSWER: 6.17%
   (b) HINT: Solve $1400 = P(1.03)^{11}$ for P.
   ANSWER: \$1011.39
   (c) HINT: You want to compute the value of t such that $P\left(1+\frac{0.06}{12}\right)^{12t} = 2P$. Divide both sides by P and the P's cancel. Solve the resulting equation for t using the natural logarithm.
   ANSWER: $t = \frac{\ln 2}{12\ln(1.005)} = 11.58$ years
   (d) ANSWER: \$4161.84

2. (a) HINT: Solve $1.70 = 0.55(1+r)^{11}$ for r.
   ANSWER: 10.80%
   (b) ANSWER: $450(1.03)^{21/12} = \$473.89$
   (c) HINT: Solve $P + 13 = P\left[e^{(0.0175)(1/12)}\right]$ for P.
   ANSWER: $P = \$8907.79$
   (d) HINT: Let $A(t)$ be the value of the refrigerator t years after its purchase. Then, $A(0) = 4300$, $A(1) = 4300(0.75)$, $A(2) = 4300(0.75)^2$, etc. (This is a multiplicative sequence with multiplier 0.75.) In general, $A(t) = 4300(0.75)^t$. Solve $2000 = 4300(0.75)^t$ for t. (Start by dividing both sides by 4300 and then take the natural log of both sides.)
   ANSWER: $t = 2.66$ years

3. (a) ANSWER: Compute the difference between consecutive terms. If the difference is always the same, then the sequence is additive.
   (b) ANSWER: $A(k+1) = A(k) + 5$
   (c) HINT: You are told that $B(k)$ is additive. Its increment is the difference between any two consecutive terms. So, look at $B(2) - B(1)$:

   $$B(2) - B(1) = [2A(2) + 4] - [2A(1) + 4] = 2[A(2) - A(1)].$$

   Since the increment of sequence A is 5, $A(2) - A(1) = 5$.
   ANSWER: 10
   (d) HINT: $B(k) = 22 + 10(k-1)$
   ANSWER: $B(108) = 1092$

4. (a) HINT: Set $MR = MC$ and solve for q using the quadratic formula.
   ANSWER: $q = 5$ and $q = 20$
   (b) ANSWER: From the graph of MR and MC, we can see that profit is maximized at $q = 20$, because that is the quantity at which there is a transition from $MR > MC$ (profit is increasing) to $MR < MC$ (profit is decreasing). At $q = 5$, we see the opposite transition.
   (c) ANSWER: $TR(q) = -0.01q^3 + 0.71q^2 + 4q$, $VC(q) = 0.01q^3 - 0.04q^2 + 10q$
   (d) HINT: $TC(q) = VC(q) + FC = 0.01q^3 - 0.04q^2 + 10q + FC$. Since $TC(6) = 100$, solve $100 = 0.01(6)^3 - 0.04(6)^2 + 10(6) + FC$ for FC.
   ANSWER: $FC = \$39.28$
   (e) ANSWER: $0.01q^3 - 0.04q^2 + 10q = -0.01q^3 + 0.71q^2 + 4q - 120$ (or any equivalent equation)

5. (a) HINT: Each player starts with 1000 points. Jessica averages 70 points per minute in the first 6 minutes ($t = 0$ to $t = 6$), 30 points per minute in the next 6 minutes ($t = 6$ to $t = 12$), and 0 points per minute in the next 6 minutes ($t = 12$ to $t = 18$). So, Jessica's score after 18 minutes is $J(18) = 1000 + 70(6) + 30(6) + 0(6) = 1600$. You can compute Robert's score at 18 minutes in a similar fashion.

   ANSWER: Jessica is ahead by 660 points.

   (b) HINT: If Robert's score increased by 300 points in a 6-minute period, then his incremental rate of change during that six minutes was $\frac{300}{6} = 50$.

   ANSWER: from $t = 24$ to $t = 30$

   (c) ANSWER: between $t = 42$ and $t = 48$ minutes

6. (a) ANSWER: $t = 0.95, 2.4, 3.35$ (all approximations from the graph)

   (b) HINT: You're computing the slope of the secant line through the graph of $A(t)$ at $t = 0$ and $t = 3$.

   ANSWER: approximately 750 thousand gallons per hour

   (c) HINT: The incremental rate of change from $t = 1$ to $t = 1.01$ is the slope of the secant line through the graph of $A(t)$ at $t = 1$ and $t = 1.01$. We approximate this slope by computing the slope of the line tangent to $A(t)$ at $t = 1$.

   ANSWER: approximately 1.67 thousand gallons per hour (any answer between 1.5 and 2.0 is acceptable)