

MATH 111 B – Version 1
Final Exam
December 17, 2002

Name _____

Student ID # _____

Section _____

1	16	
2	20	
3	16	
4	23	
5	13	
6	12	
Total	100	

- You are allowed to use a calculator, a ruler, and one sheet of handwritten notes.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit.
- If you use a trial and error method when an algebraic method is available, you will not receive full credit.
- Write your answers in the specified locations.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so. If you still need more paper, please ask for some.
- Raise your hand if you have a question.
- You have 1 hour and 50 minutes to complete the exam.

GOOD LUCK!

1. (16 points) You are considering putting money into one of two accounts. The nominal period is one year.

Account A: nominal rate 5%, compounded quarterly

Account B: nominal rate 8%, compounded continuously

In each of the following questions, round your **final answer** to two digits after the decimal.

- (a) (4 points) Suppose you deposit \$100 into account *A*. What is the percentage change in your balance after 1 year?

ANSWER: _____%

- (b) (4 points) Compute the future value of \$1100 to be received from Account *B* 14 years from now.

ANSWER: \$_____

- (c) (4 points) Compute the length of time it would take to triple your principal in Account *A*.

ANSWER: _____ years

- (d) (4 points) Suppose Account *B* is not available to you now, but it will be in 5 years. You deposit \$3000 in Account *A* for 5 years and then deposit the resulting balance in Account *B*. What is your balance after a **total** of 12 years?

ANSWER: \$_____

2. (20 points) In each of the following questions, round your **final answer** to two digits after the decimal.

- (a) (5 points) An action-figure collector purchased a Bionic Woman action-figure in 1978. The collector sells the figure on e-Bay in 2002 for \$11.02. If the annual rate of return for the investment is 1.35%, how much did the figure cost in 1978?

ANSWER: \$ _____

- (b) (5 points) Marsha loans her daughter Julie \$650 to buy a new amplifier. Julie pays the loan back in 15 months with interest of 2% per year (compounded annually). What is the total sum that Julie pays her mother?

ANSWER: \$ _____

- (c) (5 points) Matt and Jenni would like to endow their rent. That is, they will invest some amount of money in an account and use the interest to pay the rent on their apartment each month. How much would they have to invest in an account paying 4.25% annual interest, compounded continuously, to earn \$1065 *in interest* after one month?

ANSWER: \$_____

- (d) (5 points) A virus spreads so that the infected population increases by 25% every hour. Suppose 10 people are infected at noon. After how many hours will 200 people be infected?

ANSWER: _____ hours

3. (16 points)

(a) (4 points) Describe how you determine whether or not a sequence is multiplicative.

(b) (4 points) Suppose $A(k)$ is a multiplicative sequence with multiplier 9. Give an equation that relates $A(k+1)$ to $A(k)$.

ANSWER: _____

(c) (4 points) Let $A(k)$ be the sequence described in part (b). Let $B(k)$ be a sequence with the following property:

$$B(1) = 2\sqrt{A(1)}, B(2) = 2\sqrt{A(2)}, B(3) = 2\sqrt{A(3)}, \dots$$

$B(k)$ is also a multiplicative sequence. Find its multiplier.

(HINT: You may need to use the fact that $\frac{\sqrt{x}}{\sqrt{y}} = \sqrt{\frac{x}{y}}$ for any numbers x and y .)

ANSWER: _____

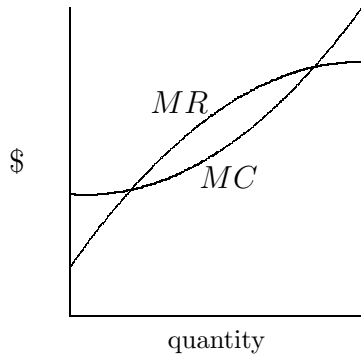
(d) (4 points) Let $B(k)$ be the sequence described in part (c). Suppose $B(2) = 990$. Give an explicit formula for $B(k)$.

ANSWER: $B(k) =$ _____

4. (23 points) You sell items. The Marginal Revenue and Marginal Cost are given by the formulas:

$$MR(q) = -0.03q^2 + 1.42q + 4 \text{ and } MC(q) = 0.03q^2 - 0.08q + 10.$$

The quantity is in items and the MR and MC are in dollars. The graphs of MR and MC are given below.



- (a) (6 points) Find the two quantities at which $MR = MC$.

ANSWER: $q =$ _____ and $q =$ _____

- (b) (5 points) Profit is maximized at one of the quantities you found in part (a) but not at the other. Decide which quantity maximizes profit and give a reason for your choice.

ANSWER: Profit is maximized when $q =$ _____ items because:

We are still considering the functions

$$MR(q) = -0.03q^2 + 1.42q + 4 \text{ and } MC(q) = 0.03q^2 - 0.08q + 10.$$

(c) (4 points) If $MR = aq^2 + bq + c$, then the formula for Total Revenue is

$$TR = \frac{a}{3}q^3 + \frac{b}{2}q^2 + cq.$$

If $MC = aq^2 + bq + c$, then the formula for Variable Cost is

$$VC = \frac{a}{3}q^3 + \frac{b}{2}q^2 + cq.$$

Give the formulas for Total Revenue and Variable Cost for your items.

ANSWER: $TR(q) =$ _____

$VC(q) =$ _____

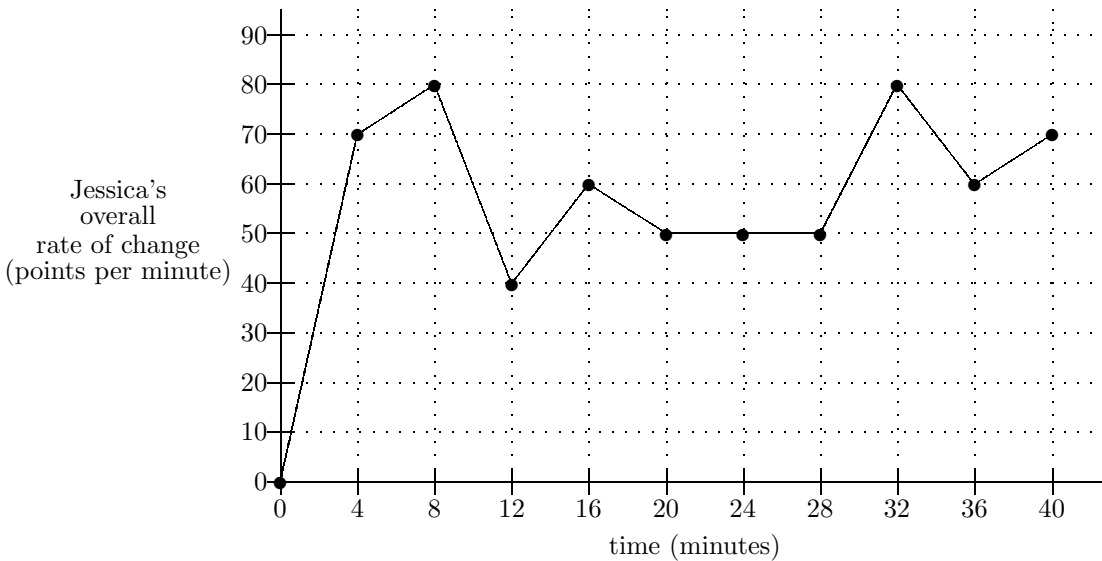
(d) (5 points) Which value of Fixed Cost will make the Total Cost of producing 9 items \$100?

ANSWER: $FC = \$$ _____

(e) (3 points) Set up an equation that you would solve in order to find the quantity at which Variable Cost is \$120 less than Total Revenue. Do not attempt to solve the equation.

ANSWER: _____

5. (13 points) Jessica and Robert are playing a game in which they can earn and lose points. At the start of the game, each player has 0 points. The following is the graph of the *overall rate of change* in Jessica's score computed every 4 minutes.



The overall rate of change of Robert's score is **always** 70 points per minute. Let $J(t)$ represent Jessica's score at time t and $R(t)$ denote Robert's score at time t .

- (a) (5 points) Which player is ahead after 8 minutes? By how much?

ANSWER: (circle one) [Jessica Robert] is ahead by _____ points

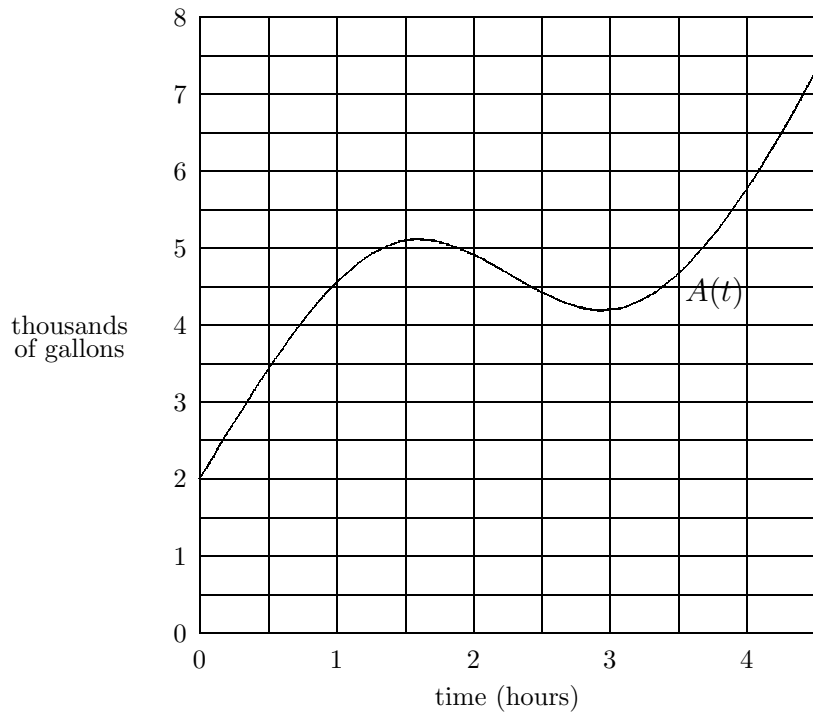
- (b) (4 points) What is the *incremental* rate of change of Jessica's score from $t = 12$ to $t = 16$?

ANSWER: _____ points per minute

- (c) (4 points) Name the time at which Robert's score is 1000 points. Round the time to the nearest minute.

ANSWER: $t =$ _____ minutes

6. (12 points) Below is the graph of the amount of water in a tank at time t . We denote the amount of water in the tank by $A(t)$.



- (a) (4 points) Find all times at which there are 4,500 gallons in the tank.

ANSWER: $t =$ _____

- (b) (4 points) The overall average rate of change of the amount of water in the tank is given by

$$R(t) = \frac{A(t) - A(0)}{t}.$$

Compute the overall average rate at $t = 3$. Round your answer to two digits after the decimal, if necessary.

ANSWER: $R(3) =$ _____ thousand gallons per hour

- (c) (4 points) Estimate the *incremental* rate of change in the amount of water over the time interval $t = 1$ to $t = 1.01$ hours. Round your answer to two digits after the decimal, if necessary.

ANSWER: _____ thousand gallons per hour