MATH 111B – Autumn 2002
Final Exam
Hints and Answers

1. (a) HINT: You earn \$5.09 in interest in your first year. What percentage of your original deposit is this?
   ANSWER: 5.09%
   (b) ANSWER: $1100e^{(0.08)(14)} = \$3371.34$
   (c) HINT: You want to compute the value of t such that $P\left(1+\frac{0.05}{4}\right)^{4t} = 3P$ for t. Divide both sides by P and the P's cancel. Solve the resulting equation for t using the natural logarithm.
   ANSWER: $t = \frac{\ln 3}{4\ln(1.0125)} = 22.11$ years
   (d) ANSWER: \$6733.28

2. (a) HINT: Solve $11.02 = P(1.0135)^{24}$ for P.
   ANSWER: \$7.99
   (b) ANSWER: $650(1.02)^{15/12} = 666.29$
   (c) HINT: Solve $P + 1065 = P\left[e^{(0.0425)(1/12)}\right]$ for P.
   ANSWER: $P = \$300,173.69$
   (d) HINT: Solve $200 = 10(1.25)^t$ for t. (Start by dividing both sides by 10 and then take the natural log of both sides.)
   ANSWER: $t = 13.43$ years

3. (a) ANSWER: Compute the ratio of consecutive terms. If this ratio is always the same, then the sequence is multiplicative.
   (b) ANSWER: $A(k+1) = 9A(k)$
   (c) HINT: You are told that $B(k)$ is a multiplicative sequence. Its multiplier is the ratio of any two consecutive terms.

$$\frac{B(2)}{B(1)} = \frac{2\sqrt{A(2)}}{2\sqrt{A(1)}} = \sqrt{\frac{A(2)}{A(1)}}.$$

   Since A is a multiplicative sequence with multiplier 9, $\frac{A(2)}{A(1)} = 9$.
   ANSWER: 3
   (d) HINT: Use the fact that $B(2) = 3B(1)$ to compute the value of $B(1)$. Then, the explicit formula is $B(k) = 3^{(k-1)}B(1)$.
   ANSWER: $B(k) = 3^{(k-1)} \cdot 330$

4. (a) HINT: Set $MR = MC$ and solve for q using the quadratic formula.
   ANSWER: $q = 5$ and $q = 20$
   (b) ANSWER: From the graph of MR and MC, we can see that profit is maximized at $q = 20$, because that is the quantity at which there is a transition from $MR > MC$ (profit is increasing) to $MR < MC$ (profit is decreasing). At $q = 5$, we see the opposite transition.
   (c) ANSWER: $TR(q) = -0.01q^3 + 0.71q^2 + 4q$, $VC(q) = 0.01q^3 - 0.04q^2 + 10q$
   (d) HINT: $TC(q) = VC(q) + FC = 0.01q^3 - 0.04q^2 + 10q + FC$. Since $TC(9) = 100$, solve $100 = 0.01(9)^3 - 0.04(9)^2 + 10(9) + FC$ for FC.
   ANSWER: $FC = \$5.95$
   (e) ANSWER: $0.01q^3 - 0.04q^2 + 10q = -0.01q^3 + 0.71q^2 + 4q - 120$ (or any equivalent equation)

5. (a) HINT: $\frac{J(8)}{8} = 80$ and $\frac{R(8)}{8} = 70$. Compute $J(8)$ and $R(8)$ and subtract.
   ANSWER: Jessica is ahead by 80 points.

(b) HINT: $\frac{J(12)}{12} = 40$ and $\frac{J(16)}{16} = 60$. You need to compute $\frac{J(16)-J(12)}{4}$.
ANSWER: 120 points per minute.

(c) HINT: You are told that $\frac{R(t)}{t} = 70$ for every value of t. You want to know when $R(t) = 1000$. So, solve $\frac{1000}{t} = 70$ for t.

ANSWER: $t = 14$ minutes (rounded to the nearest minute)

6. (a) ANSWER: $t = 0.95, 2.4, 3.35$ (all approximations from the graph)

(b) HINT: You're computing the slope of the secant line through the graph of $A(t)$ at $t = 0$ and $t = 3$.

ANSWER: approximately 750 thousand gallons per hour

(c) HINT: The incremental rate of change from $t = 1$ to $t = 1.01$ is the slope of the secant line through the graph of $A(t)$ at $t = 1$ and $t = 1.01$. We approximate this slope by computing the slope of the line tangent to $A(t)$ at $t = 1$.

ANSWER: approximately 1.67 thousand gallons per hour (any answer between 1.5 and 2.0 is acceptable)