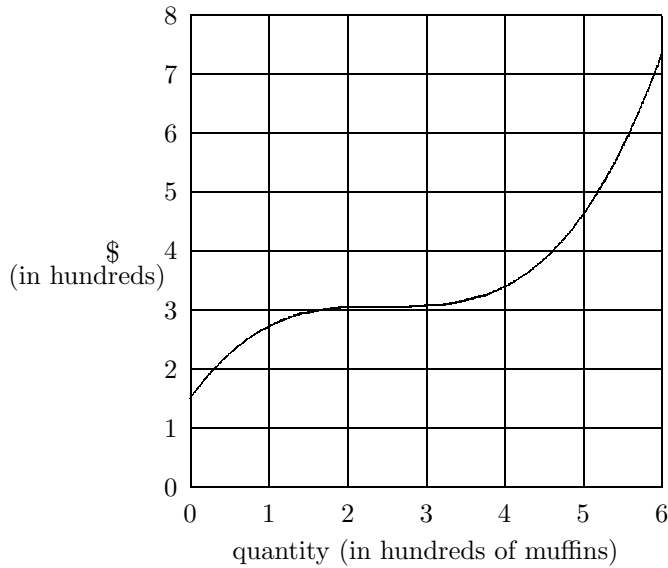


**MATH 111 A, B**  
Autumn 2002  
Final Exam Review Sheet

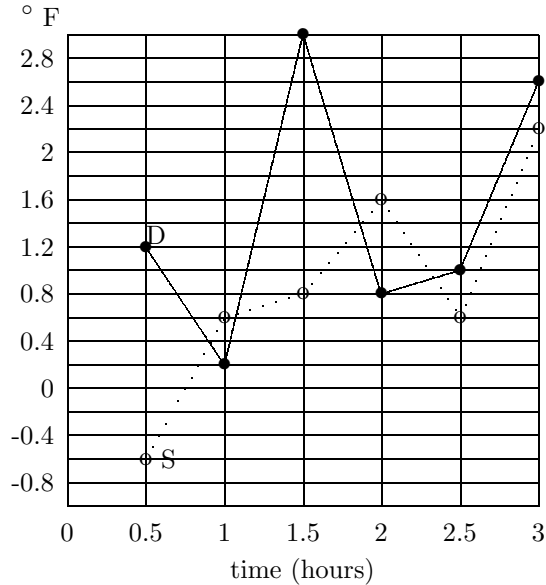
This is by no means exhaustive coverage of all the topics you might see on the final exam. You should review the Sample Exams and your Midterms. You are expected to know how to complete all assigned Pre-Test problems.

1. You produce and sell muffins. The graph below is the graph of total costs for producing any number of muffins from 0 to 600 muffins.



- (a) Carefully estimate the marginal cost at 100 muffins. Clearly indicate how you arrive at your estimate.
- (b) Carefully estimate the average cost at 100 muffins. Clearly indicate how you arrive at your estimate.
- (c) No matter how many muffins you make, they will sell for \$1 per muffin. Sketch the graph of total revenue versus quantity in the coordinate system above. Carefully estimate the number of muffins at which total revenue equals total cost.
- (d) Using the  $TR$  graph from part (c), carefully estimate the number of muffins at which marginal revenue equals marginal cost. (You should get two different quantities.)
- (e) Profit is maximized at only one of the quantities you found in part (d). Which quantity maximizes profit? Give two reasons for your choice:
  - i. one reason involving the values of  $TR$  and  $TC$ ;
  - ii. one reason involving the values of  $MR$  and  $MC$ .
- (f) You decide to drop the price of your muffins. What is the lowest price at which you won't be forced to take a loss? (That is, for any lower price, you lose money no matter how many muffins you sell.)

2. The temperature in Denver at 6 a.m. one recent morning was  $17^\circ$  F. The temperature in Seattle on the same morning at 6 a.m. was  $T^\circ$  F. The graph shows the **change** in the temperature of each city every half hour after 6 a.m. For example, at 6:30 a.m. the temperature in Denver has risen  $1.2^\circ$  to  $18.2^\circ$ .



- (a) What is the temperature in Denver at 8 a.m.?
- (b) In Seattle, when is the temperature  $0.8^\circ$  **more** than the temperature at 6 a.m.?
- (c) Let  $P(t)$  be the temperature in Seattle  $t$  hours after 6 a.m. What is the value of  $P(2.5) - P(1)$ ?
3. You sell lint-lumps. The formulas for Total Revenue and Total Cost (both in dollars) are as follows.

$$TR : R(q) = -\frac{1}{5}q^2 + 81q \quad TC : C(q) = 15q + 75$$

- (a) Compute the quantity that yields the maximum possible profit. What is the maximum possible profit?
- (b) Recall that Marginal Revenue is given by  $MR = R(q+1) - R(q)$  and Marginal Cost is given by  $MC = C(q+1) - C(q)$ . Use these formulas to find the Marginal Revenue and Marginal Cost at  $q$ .
- (c) Find the quantity  $q$  at which Marginal Revenue and Marginal Cost are equal.
- (d) Without actually computing  $MR$  and  $MC$  at  $q = 164$ , use parts (a) and (c) to determine whether  $MR < MC$  or  $MR > MC$  at  $q = 164$ . Explain your answer.
- (e) Find the quantity  $q$  at which MR exceeds MC by \$17.
- (f) Find the quantity  $q$  at which average total cost (AC) is \$20 per lint-lump.

4. You have money in accounts I and II.
- Account I:** 11% interest, compounded quarterly
- Account II:** interest rate  $r$ , compounded continuously
- (a) What is the value now of a deposit made to account I that will be worth \$5 in 14 months?
- (b) What interest rate  $r$  for account II will give the two accounts the same annual percentage yield (APY)?
5. Theaters are often built with more seats per row as the rows move toward the back. Suppose that the main floor of a theater has 28 seats in the first row, 32 in the second, 36 in the third, and so on, for 50 rows. How many seats are in the last row?
6. Consider the sequence  $A(1) = x + 3$ ,  $A(2) = x + 7$ ,  $A(3) = 4x - 2$ , ... Suppose the sequence is arithmetic with increment  $I$ .
- (a) How is  $A(2)$  related to  $A(1)$ ? (Your answer should involve  $I$ .)
- (b) Use your answer to (a) to find  $I$ .
- (c) How is  $A(3)$  related to  $A(2)$ ?
- (d) Use your answer to (c) to find  $x$ .
- (e) Find  $A(4)$ . (Your answer should be a number.)
7. An office machine is purchased for \$5200. Its value each year is about 80% of the value the preceding year.
- (a) Find an explicit formula for  $V(t)$ , the value of the machine after  $t$  years.
- (b) After what amount of time will the salvage value be \$1200?
- (c) After what amount of time will the salvage value be half of its original value?
8. (a) What is the future value, 15 years from now, of \$100,000 in an account paying 8% interest, compounded quarterly.
- (b) What is the present value of \$7000, 15 years from now, in an account paying 5.2% interest, compounded monthly?
- (c) What is the smallest interest rate that will allow an initial deposit of \$14,500 to yield \$25,000 in 11 years if interest is compounded semi-annually?
- (d) What is the smallest interest rate that will allow an initial deposit of \$14,500 to yield \$25,000 in 11 years if interest is compounded continuously?
- (e) How long would it take to triple an investment with 8.3% interest, compounded annually?
- (f) How long would it take to quadruple an investment with 5.9% interest, compounded continuously?
- (g) How much would you have to invest in an account paying 6% interest, compounded semi-annually to earn \$50 *in interest* in 9 months?

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Final Exam Review Sheet Answers

1. All answers are approximated from the graph.
  - (a)  $MC(100) = \$0.65$
  - (b)  $AC(100) = \$2.75$  per muffin
  - (c)  $TR = TC$  at  $q = 3.1$  (310 muffins) or  $q = 5.3$  (530 muffins)
  - (d)  $MR = MC$  at  $q = 0.60$  (60 muffins) or  $q = 4.3$  (430 muffins)
  - (e) Profit is maximized when you sell 430 muffins but not when you sell 60 muffins because  $MR = MC$  at that quantity AND:
    - i.  $TR > TC$  at 430 muffins but  $TR < TC$  at 60 muffins — we're taking a loss at 60 muffins.
    - ii. For quantities to the left of  $q = 0.6$ ,  $MC > MR$  (look at the tangent lines). So, profit is decreasing there. For quantities to the right of  $q = 0.6$ ,  $MC < MR$ . So, profit is increasing there. Profit changes from decreasing to increasing at  $q = 0.6$ . That means profit is at a minimum at  $q = 0.6$ , not a maximum. On the other hand,  $MR > MC$  for quantities just to the left of  $q = 4.3$  and  $MR < MC$  for quantities just to the right of  $q = 4.3$ . So, profit is changing from increasing to decreasing at  $q = 4.3$ . So, that's where profit is maximized.
  - (f)  $p = \$0.86$  per muffin
2.
  - (a)  $22.2^\circ$  F
  - (b) 7:30
  - (c)  $3.0^\circ$  F
3.
  - (a) The maximum possible profit is \$5370. You get max profit when  $q = 165$ .
  - (b)  $MR(q) = -0.4q + 80.8$  and  $MC(q) = 15$
  - (c)  $MR = MC$  at  $q = 164.5$
  - (d) Since profit is maximized at  $q = 165$ , this is the place where things change from  $MR > MC$  to  $MR < MC$ . Since  $q = 164$  is to the left of  $q = 165$ , it must be the case that  $MR > MC$  (profit is increasing) at  $q = 164$ .
  - (e)  $MR = MC + 17$  at  $q = 122$
  - (f)  $AC = 20$  at  $q = 15$
4.
  - (a) \$4.41
  - (b) 10.85%
5. 224 seats

6. (a)  $A(2) = A(1) + I$   
(b)  $I = 4$   
(c)  $A(3) = A(2) + 4$   
(d)  $x = \frac{13}{3}$   
(e)  $A(4) = \frac{58}{3}$
7. (a)  $V(t) = 5200(0.80)^t$   
(b)  $t = 6.57$  years  
(c)  $t = 3.11$  years
8. (a) \$328,103.08  
(b) \$3214.25  
(c) 5.01%  
(d) 4.95%  
(e) 13.78 years  
(f) 23.50 years  
(g) \$1102.88