

MATH 111  
Final Exam  
December 13, 2003

Name \_\_\_\_\_

Signature \_\_\_\_\_

Student ID # \_\_\_\_\_

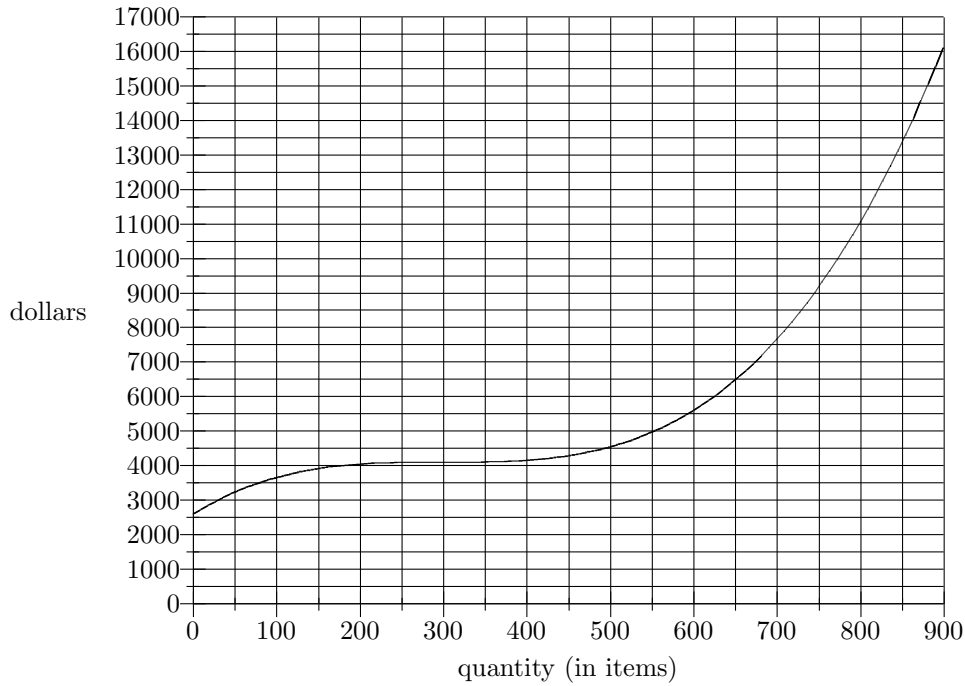
Section \_\_\_\_\_

1	9	
2	12	
3	15	
4	14	
5	20	
6	30	
Total	100	

- You are allowed to use a calculator, a ruler, and one sheet of handwritten notes.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit.
- Write your answers in the specified locations. Unless otherwise indicated, you may round your **final answer** to two digits after the decimal.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so. If you still need more paper, please ask for some.
- Raise your hand if you have a question.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have three hours to complete the exam.

GOOD LUCK!

1. (9 points) You sell *items* in a store. The graph below is of total cost ( $TC$ ).



(a) (3 points) Find the break-even price.

ANSWER: \$ \_\_\_\_\_

(b) (2 points) You can sell your items for \$15 each. Draw the graph of total revenue. Label it  $TR$ .

(c) (2 points) Find the smallest quantity at which you break even.

ANSWER:  $q =$  \_\_\_\_\_ items

(d) (2 points) What quantity do you need to sell in order to maximize profit?

ANSWER:  $q =$  \_\_\_\_\_ items

2. (12 points) You sell *things*. Your total cost,  $TC$ , is given in dollars as a function of quantity  $q$  by

$$TC(q) = q^3 - 9q^2 + 30q + 100.$$

- (a) (2 points) What is your fixed Cost,  $FC$ ?

ANSWER:  $FC = \$$ \_\_\_\_\_

- (b) (4 points) What is your marginal cost ( $MC$ ) at  $q = 5$ ? (HINT: You can answer this question *without* finding a general formula for  $MC(q)$ .)

ANSWER:  $MC(5) = \$$ \_\_\_\_\_

- (c) (2 points) Find a formula for average cost ( $AC$ ).

ANSWER:  $AC(q) =$ \_\_\_\_\_

- (d) (4 points) What is the largest value of  $q$  for which average variable cost ( $AVC$ ) is \$22 per thing?

ANSWER:  $q =$ \_\_\_\_\_ things

3. (15 points) A red car travels along a long, straight road. After  $t$  minutes, the **distance** the red car has traveled (in miles) is  $R(t) = 1.5t - 0.1t^2$ .

- (a) (3 points) Find the linear formula for the red car's average trip speed at  $t$  minutes. Simplify your formula and put it in the form  $b + mt$ .

ANSWER: average trip speed =  $(\quad) + (\quad)t$

- (b) (3 points) How far has the red car traveled when its average trip speed is 0.8 mpm?

ANSWER: \_\_\_\_\_ miles

- (c) (5 points) The red car's average (incremental speed) over a 2-minute interval starting at time  $t$  is given by

$$\frac{R(t+2) - R(t)}{2}.$$

Find the 2-minute interval over which the car's average speed is 0.75 mpm.

ANSWER: from  $t =$  \_\_\_\_\_ to  $t =$  \_\_\_\_\_

- (d) (4 points) A green car has distance formula

$$G(t) = 0.2t + 0.05t^2, \quad (t \text{ in minutes, } G(t) \text{ in miles}).$$

How far has the red car gone when the green car has gone 4 miles?

ANSWER: \_\_\_\_\_ miles

4. (14 points) Two quantities vary with time  $t$ :

$$f(t) = t^2 - 10t + 24 \text{ and } g(t) = -t^2 + 4t.$$

- (a) (2 points) Find the time when  $f(t)$  is at its minimum.

ANSWER:  $t =$  \_\_\_\_\_

- (b) (4 points) Find the largest value of  $g(t)$ .

ANSWER: \_\_\_\_\_

- (c) (3 points) Find the earliest time at which  $f(t)$  and  $g(t)$  are the same.

ANSWER:  $t =$  \_\_\_\_\_

- (d) (2 points) For what range of time is  $g(t) > f(t)$ ?

ANSWER: from  $t =$  \_\_\_\_\_ to  $t =$  \_\_\_\_\_

- (e) (3 points) At what time is the difference  $(g(t) - f(t))$  between the amounts the greatest?

ANSWER:  $t =$  \_\_\_\_\_

5. (20 points – 5 points each) Lenny and Carl both have bank accounts. Lenny's earns 7% per year, compounded annually. Carl's account earns 6.5% per year, compounded quarterly.

(a) If Carl deposits \$9000 in his account at  $t = 0$ , what is his balance after 10 years?

ANSWER: \$\_\_\_\_\_

(b) How much should Lenny deposit at  $t = 0$  in order to have a balance of \$20,000 after 15 years?

ANSWER: \$\_\_\_\_\_

(c) If Lenny puts \$10,000 in his account at  $t = 0$ , how much should Carl put in his account at  $t = 0$  so that they have identical balances at  $t = 6$  years?

ANSWER: \$\_\_\_\_\_

(d) If Lenny and Carl each deposit \$4500 in their respective accounts at  $t = 0$ , then how much will Lenny have in his account when Carl has \$12,000?

ANSWER: \$\_\_\_\_\_

6. (30 points – 5 points each) You are choosing between four accounts.

**Account A:** 12% compounded annually

**Account B:** 12% compounded monthly

**Account C:** 12% compounded continuously

**Account D:**  $(r \times 100)\%$  compounded monthly (That is,  $r$  is the interest rate, expressed as a decimal.)

- (a) How much interest will you earn after 14 months if you deposit \$2000 into account  $B$ ?

ANSWER: \$\_\_\_\_\_

- (b) What is the value of one dollar, 7 months after it has been placed in account  $C$ ?

ANSWER: \$\_\_\_\_\_

- (c) What is the present value of \$3900 to be received from account  $A$ , 18 months from now?

ANSWER: \$\_\_\_\_\_

- (d) What is the time required for \$2000 to be increased by 40% in account  $C$ ?

ANSWER: \_\_\_\_\_ years

- (e) You deposit \$1000 into account  $D$ . After 3 years, the account contains \$2000. Find the value of  $r$ .

ANSWER:  $r =$  \_\_\_\_\_

- (f) You deposit \$1000 into account  $B$ . You want to withdraw a certain amount of money from the account at the end of each month, but you want to make the same withdrawal FOREVER. What is the largest amount of money you can withdraw at the end of every month so that you could continue doing it forever?

ANSWER: \$ \_\_\_\_\_