MATH 111
Autumn 2003 – Final Exam
Hints and Answers

1. (a) ANSWER: Something around \$9.
   (b) HINT: Draw a line through the origin with slope 15.
   (c) ANSWER: Something around 260 items.
   (d) HINT: Find where the tangent line to TC is parallel to TR.
   ANSWER: Something around 600 items.

2. (a) ANSWER: \$100
   (b) HINT: $MC(5) = TC(6) - TC(5)$
   ANSWER: \$22
   (c) ANSWER: $AC(q) = \frac{TC(q)}{q} = q^2 - 9q + 30 + \frac{100}{q}$
   (d) HINT: $VC(q) = q^3 - 9q^2 + 30q$ and $AVC(q) = q^2 - 9q + 30$. Set $AVC = 22$ and solve for q.
   ANSWER: $q = 8$

3. (a) ANSWER: ats $= \frac{R(t)}{t} = 1.5 - 0.1t$
   (b) HINT: Set ats from part (a) equal to 0.8 and solve for t ($t = 7$). Then compute $R(7)$.
   ANSWER: 5.6 miles
   (c) HINT: Compute and simplify the formula for average speed over a 2-minute interval:
   $$\frac{R(t+2) - R(t)}{2} = 1.3 - 0.2t.$$
   Then set this equal to 0.75 and solve for t.
   ANSWER: from $t = 2.75$ to $t = 4.75$
   (d) HINT: Set $G(t) = 4$ and solve for t ($t = 7.1651514$). Then compute $R(7.1651514$.
   ANSWER: 5.61 miles

4. (a) HINT: $f(t)$ is a quadratic function whose graph is a parabola that opens up. Its minimum occurs at the vertex.
   ANSWER: $t = 5$
   (b) HINT: $g(t)$ is a quadratic function whose graph is a parabola that opens down. Its largest value occurs at the vertex. The vertex formula gives the t-coordinate of the vertex ($t = 2$). So, the largest value of $g(t)$ is $g(2)$.
   ANSWER: 4
   (c) HINT: Set $f(t)$ equal to $g(t)$ and solve for t.
   ANSWER: $t = 3$
   (d) HINT: Think about the graphs of the two functions and where they intersect (part (c)).
   ANSWER: from $t = 3$ to $t = 4$
   (e) HINT: $g(t) - f(t) = -2t^2 + 14t - 24$. This is a quadratic function whose graph is a parabola that opens down. It is greatest at the vertex.
   ANSWER: $t = 3.5$

5. HINT: Let $L(t)$ and $C(t)$ be the amount in Lenny and Carl's respective bank accounts after t years. Then $L(t) = P(1.07)^t$ and $C(t) = P(1.01625)^{4t}$.

   (a) HINT: Take $P = 9000$ and compute $C(10)$.
   ANSWER: \$17,150.03
   (b) HINT: Set $L(15) = 20,000$ and solve for P.
   ANSWER: \$7248.92

(c) HINT: Take $P = 10,000$ and compute $L(6)$ ($L(6) = 15,007.30$). Then set $C(6) = 15,007.30$ and solve for P.

ANSWER: \$10,192.70

(d) HINT: Take $P = 4500$ and set $C(t) = 12,000$ and solve for t (using the natural logarithm). You should get that $t = 15.211955$. Then, take $P = 4500$ again and compute $L(15.211955)$.

ANSWER: \$12,594.97

6. (a) HINT: The amount in account B after t years is $B(t) = P(1.01)^{12t}$. Take $P = 2000$ and compute $B\left(\frac{14}{12}\right)$. Then, subtract off the \$2000 principal to get the earned interest.

ANSWER: \$298.95

(b) HINT: The amount in account C after t years is $C(t) = Pe^{0.12t}$. Take $P = 1$ and compute $C\left(\frac{7}{12}\right)$.

ANSWER: \$1.07

(c) HINT: The amount in account A after t years is $A(t) = P(1.12)^{1.15}$. Set $A(1.5)$ equal to 3900 and solve for P.

ANSWER: \$3290.32

(d) HINT: Forty percent of \$2000 is \$800. So, take $P = 2000$ and set $C(t)$ equal to \$2800 and solve for t (using the natural logarithm).

ANSWER: $t = 2.80$ years

(e) HINT: The amount in account D after t years is $D(t) = P\left(1 + \frac{r}{12}\right)^{12t}$. Take $P = 1000$, set $D(3) = 2000$, and solve for r.

ANSWER: $r = 0.2333$

(f) HINT: You will be able to make the same withdrawl every month if you withdraw only the interest earned each month. So, take $P = 1000$ and compute $A\left(\frac{1}{12}\right)$ (\$1010). You therefore earn \$10 in interest in one month. If you withdraw this \$10, then you will make \$10 again the next month and so on.

ANSWER: \$10