MATH 111 – Final Exam Hints and Answers
Autumn 2005

1. (a) HINT: Solve for P: $1 = P\left(1 + \dfrac{0.071}{4}\right)^{4(10)}$.

   ANSWER: \$0.49

   (b) HINT: $APY = \left[\left(1 + \dfrac{0.069}{3}\right)^{3} - 1\right] \times 100\%$

   ANSWER: APY = 7.06%

   (c) HINT: OLD = P; NEW = $P\left(1 + \dfrac{0.054}{12}\right)^{12(1/2)}$; percent change = $\frac{\text{NEW}-\text{OLD}}{\text{OLD}} \times 100\%$

   ANSWER: 2.73%

   (d) HINT: Solve for t: $7220 = 1700\left(1 + \dfrac{0.047}{2}\right)^{2t}$

   ANSWER: 31.13 years

2. (a) HINT: $A(4) = Pe^{4r}$; $A(4.75) = Pe^{4.75r}$; $A(4.75) = X \cdot A(4)$. Put all of that together and solve for X.
   ANSWER: $e^{0.75r}$

   (b) HINT: Solve for r: $4P = Pe^{12r}$.
   ANSWER: $r = 0.1155$

   (c) HINT: Solve for r: $\left[\left(1 + \dfrac{r}{6}\right)^{6} - 1\right] \times 100\% = 3.45\%$
   ANSWER: $r = 0.0340$

   (d) HINT: In one year, the amount in account B will be \$5229.26, which means that you earn \$229.26 in interest in a year. Half that interest would be \$114.63. To find how long it takes to earn that much interest, solve the following for t:

$$5114.63 = 5000\left(1 + \frac{0.045}{6}\right)^{6t}.$$

   ANSWER: interest = \$229.26; $t = 0.5056$ years

3. (a) ANSWER: $S(k+1) - S(k) = 7$ (or anything equivalent)

   (b) HINT: $T(1) - T(0) = [6S(1) - 5] - [6S(0) - 5] = 6S(1) - 5 - 6S(0) + 5 = 6[S(1) - S(0)] = 6(7) = 42$ and $T(2) - T(1) = [6S(2) - 5] - [6S(1) - 5] = 6[S(2) - S(1)] = 6(7) = 42$.
   ANSWER: yes; increment = 42

   (c) HINT: Explicit formula: $T(k) = T(0) + 42k$. So, solve $T(17) = T(0) + 42(17)$ for $T(0)$ to get that $T(0) = 36$. Then $T(200) = 36 + 42(200) = 8436$.
   ANSWER: $T(200) = 8436$

4. (a) HINT: $TC(q) = 30 + 14q - 2q^2$ and $FC = TC(0)$
   ANSWER: 30 dollars

   (b) HINT: $VC(q) = 14q - 2q^2$ and $AVC(q) = 14 - 2q$. Solve for q: $14 - 2q = 10$.
   ANSWER: $q = 2$ things

   (c) HINT: $MR(3) = TR(4) - TR(3)$
   ANSWER: 39 dollars

(d) HINT: $P(q) = TR(q) - TC(q) = -q^2 + 46q - 30$. The graph of profit is a parabola that opens downward. The maximum occurs at the vertex.
ANSWER: $q = 23$ things

5. (a) HINT: You need to find the time t at which $A(t) - B(t) = 50$. Substitute in the formulas for $A(t)$ and $B(t)$ and apply the quadratic formula to the resulting equation.
ANSWER: $t = 1.051$ seconds and $t = 20.688$ seconds

(b) HINT: Compute and simplify $\dfrac{A(t+5) - A(t)}{5}$.
ANSWER: Average Speed $= -2t + 45$

(c) HINT: The distance between them is $A(t) - B(t) = -2.3t^2 + 50t$. This is a quadratic whose graph is a parabola that opens downward. The maximum distance between them is the "y"-coordinate of the vertex of this parabola.
ANSWER: 221.74 feet

(d) HINT: A's average trip speed is $-t + 50$. Solve for t: $-t + 50 = 40$.
ANSWER: $t = 10$ seconds

6. (a) HINT: The TR graph would be a diagonal line with slope 70. Draw this line and see where it intersects the graph of TC.
ANSWER: $q = 320$ Gizmos

(b) HINT: $TC(700)$ is approximately \$25,000. If profit at $q = 700$ is \$5,000, then this means that $TR(700)$ must be \$30,000. If Gizmos all sell for the same market price, then the TR graph is the diagonal line through the point $(700, 30000)$ and the market price is the slope of that line.
ANSWER: \$42.86

(c) ANSWER: \$12

(d) HINT: Compute the slope of the tangent line to the TC graph at $q = 100$.
ANSWER: \$27.06

7. (a) HINT: Draw a diagonal line with slope 150 and find the times at which that line intersects the Out graph.
ANSWER: $t = 8.8$ or 20.4

(b) HINT: If the reservoir is empty at midnight and is being filled at a constant rate, then the In graph is a diagonal line. You need to find the diagonal line with the smallest possible slope that allows for you never to have a shortage.
ANSWER: 197.17 gallons per hour

(c) HINT: This is the same as part (b), but now your In graph has a "y"-intercept of 3000. You need to find such a line with the smallest possible slope that allows for you never to have a shortage.
ANSWER: 20.83 gallons per hour

(d) HINT: Draw a reference line through the origin with slope 100. Find the amount of the largest shortage. That's how much you'd need to start with to avoid a shortage.
ANSWER: 1300 gallons