MATH 111 – FINAL EXAM Hints and Answers
Autumn 2006

1. (a) HINT: Solve for P: $2000000 = P\left(1+\frac{0.08}{12}\right)^{12(30)}$
   ANSWER: \$182,886.72
   (b) HINT: The number of bacteria present after k two-hour periods is $B(k) = 10000 \cdot 2^k$. Find $B(0.5)$.
   ANSWER: 14,142 bacteria
   (c) HINT: The value of the computer after t years is $A = 800(1-0.4)^t$. Find the value at $t = 1.5$.
   ANSWER: \$371.81
   (d) HINT: $A = 9(1.03)^{15}$
   ANSWER: \$14.02
   (e) HINT: $A = 10000\left(1 + 0.08 \cdot \frac{15}{12}\right)$
   ANSWER: \$11,000

2. HINT: In account A, the fee would be 10% of \$10,000: \$1,000. The remaining \$9,000 would be the principal. After t years, the balance in account A would be $9000e^{0.095t}$. After t years, the balance in account B would be $10000\left(1+\frac{0.08}{2}\right)^{2t}$.
   ANSWER: account A: \$14,472.13; account B: \$14,802.44; choose account B

3. (a) ANSWER: $D(k)$ is a multiplicative sequence with multiplier $m = 1.07$.
   (b) ANSWER: recursive: $D(k+1) = 1.07 \cdot D(k)$; explicit: $D(k) = (1.07)^k \cdot 2$
   (c) HINT: Use the explicit formula to find NEW=$D(7)$ and OLD=$D(0)$.
   ANSWER: 0.61
   (d) HINT: Solve for k: $13 = (1.07)^k \cdot 2$.
   ANSWER: 28 days
   (e) HINT: Solve for p: $13 = (1+p)^{20} \cdot 2$.
   ANSWER: 9.81%

4. (a) i. ANSWER: $TR(q) = -0.05q^2 + 12q$
   ii. ANSWER: $TC(q) = 5q + 100$
   iii. ANSWER: $P(q) = -0.05q^2 + 7q - 100$
   (b) HINT: Profit is a quadratic function whose graph is a parabola that opens downward. Profit is maximized at its vertex.
   ANSWER: \$145
   (c) HINT: Use the formula for $TR(q)$ to compute $TR(71)$ and $TR(70)$. Then, $MR(70) = TR(71) - TR(70)$.
   ANSWER: \$4.95
   (d) HINT: Set $P(q) = 0$ and solve for q.
   ANSWER: $q = 17$

5. (a) HINT: $TC(4) = FC + MC(0) + MC(1) + MC(2) + MC(3)$
   ANSWER: \$507.37

(b) HINT: Profit is increasing until MR falls below MC.
ANSWER: from $q = 0$ to $q = 8$

(c) HINT: $FC = TC(0) = 526$ and then $VC(q) = 0.001q^3 - 0.05q^2 + 2.5q$. Set $VC(q)$ equal to $TR(q)$. This will give you a cubic equation, but every term will contain a q that you can either cancel or factor out.
ANSWER: $q = 50$

(d) HINT: $AVC(q) = \frac{VC(q)}{q} = 0.001q^2 - 0.05q + 2.5$. This is a quadratic function whose graph is a parabola that opens upward. It is lowest at its vertex.
ANSWER: \$1.88

6. (a) HINT: Draw the secant line through the distance graph at $t = 25$ and $t = 35$, choose two points on the line, and compute its slope.
ANSWER: 2.33 miles per minute

(b) HINT: Draw the least steep diagonal line that intersects the distance graph, choose two points on the line (one should be the origin), and compute its slope.
ANSWER: 2.85 miles per minute

(c) HINT: Draw a diagonal line with slope 3 and find the values of t at which this line intersects the distance graph.
TRANSLATION: $\dfrac{D(t)}{t} = 3$
ANSWER: $t = 62.5$ or 87.5

(d) HINT: Draw a reference line with slope 2. Then use the rolling ruler method to find an interval of length 10 over which a secant line through the distance graph is parallel to your reference line.
TRANSLATION: The average speed over the 10-minute interval starting at t is 2 mpm.
ANSWER: $t = 27$ or 68

7. (a) HINT: Set $A(t) - B(t)$ equal to 10 and solve the resulting quadratic equation.
ANSWER: $t = 1.25$ minutes

(b) HINT: Compute $\frac{A(3)-A(1)}{3-1}$.
ANSWER: 56 feet per minute

(c) HINT: $C(t) = B(t-3) = 0.5(t-3)^2 - 10(t-3) + 75$. Expand and simplify.
ANSWER: $C(t) = 0.5t^2 - 13t + 109.5$