MATH 111 – Final Exam Hints and Answers
Autumn 2008

1. (a) HINT: Find the slope of the least steep diagonal line that intersects the TC graph.
ANSWER: approximately \$3.85 per Object

(b) HINT: $TC(8.5) = 34$ hundred dollars and $FC = 8$ hundred dollars. So, $VC(8.5) = 26$ hundred dollars and $AVC(8.5) = \dfrac{VC(8.5)}{8.5}$.
ANSWER: \$3.06 per object

(c) HINT: $TC(3) = 28$ and $28 + 14 = 42$. So you need to find a value of q such that $TC(q) = 42$ hundred dollars.
ANSWER: $q = 10.5$ hundred Objects

(d) ANSWER: from $q = 0$ to $q = 6$ hundred Objects

(e) HINT: The graph of TR is a diagonal line with slope 5.25.

i. HINT: Average revenue will equal average cost at the quantities where the graphs of TR and TC intersect.
ANSWER: $q \approx 5.9$ or $q \approx 13.2$ hundred Objects

ii. HINT: Find the largest vertical distance between TR and TC (where $TR > TC$, of course).
ANSWER: approximately 15 hundred dollars

2. (a) HINT: Set $A(t)$ equal to 7 and solve for t.
ANSWER: 3.55 minutes

(b) HINT: The car's average trip speed is $\dfrac{A(t)}{t} = -0.12t + 2.4$. Set this equal to 1.404 and solve for t.
ANSWER: $t = 8.3$ minutes

(c) HINT: The average speed over the h-minute interval beginning at $t = 5$ is $\dfrac{A(5+h) - A(5)}{h}$.
ANSWER: average speed $= -0.12h + 1.2$

(d) HINT: The graph of $B(t)$ goes through the points $(0, A(0)) = (0, 0)$ and $(10, A(10)) = (10, 12)$.
ANSWER: $B(t) = 1.2t$

(e) HINT: If you quickly sketch the graphs of $A(t)$ and $B(t)$, you'll see that car A is ahead of car B during the first 10 minutes. So, the distance between the two cars is $D(t) = A(t) - B(t)$, which is a quadratic function whose graph is a parabola that opens downward. Find the t-coordinate of its vertex.
ANSWER: $t = 5$ minutes

3. (a) HINT: Either find the "y"-coordinate of the vertex of $AVC(q)$ or set $MC = AVC$ and solve for q (using the quadratic formula) and plug that value of q back into either MC or AVC.
ANSWER: \$3 per Scrapkin

(b) HINT: Compute $MC(7)$.
ANSWER: 0.6 dollars per Scrapkin

(c) HINT: $TC(q) = AC(q) \cdot q = 0.1q^3 - 1.8q^2 + 11.1q + 12$ (in thousands of dollars) and $FC = TC(0)$.
ANSWER: $FC = 12$ thousand dollars

(d) HINT: $MR(q) = 13.5$. Set $MR = MC$ and solve for q.
ANSWER: $q = 12.63$ thousand Scrapkins

4. (a) i. ANSWER: The sequence $A(k)$ is multiplicative with multiplier $m = 1.05$.
   ii. ANSWER: The sequence $B(k)$ is additive with increment $r = 2,500$.
   iii. ANSWER: The sequence $C(k)$ is multiplicative with multiplier $m = (1.022)^2$.
   (b) HINT: $C(k) = 44000[(1.022)^2]^k$. Compute $C(10)$.
   ANSWER: \$67,994
   (c) HINT: $B(k) = 44000 + 2500k$. Set $B(k) = 59000$ and solve for k.
   ANSWER: after $k = 6$ years
   (d) HINT: $A(k) = 44000(1.05)^k$. Set $A(k) = 100000$ and solve for k to get $k = 16.82$. So, after 17 years, Anna will have a six-figure salary.
   ANSWER: in the year 2026

5. (a) HINT: $A = P \cdot 3^k$, where k is the number of two-hour periods that have elapsed. Set $1,500,000 = P \cdot 3^{7.5}$ and solve for P.
   ANSWER: 396 bacteria
   (b) $A = 5000 \cdot 3^4$.
   ANSWER: 405,000 bacteria
   (c) HINT: In twelve hours (6 two-hour periods), the population changes from an OLD value (P) to a NEW value ($P \cdot 3^6$).
   ANSWER: 72,800%
   (d) HINT: Set $4P = P \cdot 3^k$ and solve for k. This gives $k = 1.2618...$ as the number of two-hour periods it takes to quadruple the population.
   ANSWER: 2.52 hours

6. (a) ANSWER: \$110,401.98
   (b) ANSWER: \$57,120.91
   (c) ANSWER: 9.64%
   (d) HINT: The APY for Account D is approximately 9.73%.
   ANSWER: Account D is better.
   (e) HINT: The future value is $5000(1 + 0.03 \cdot 11)$.
   ANSWER: \$6,650
   (f) HINT: Set $2P = Pe^{5r}$ and solve for r.
   ANSWER: $r = 0.1386$
   (g) HINT: $A(1) = \$10,857.64$ and $A(2) = \$11,788.83$. The interest earned in the second year is $A(2) - A(1)$.
   ANSWER: \$931.19