

MATH 111  
Final Exam  
March 13, 2004

Name \_\_\_\_\_

Student ID # \_\_\_\_\_

Section \_\_\_\_\_

1	24	
2	14	
3	11	
4	24	
5	15	
6	12	
Total	100	

- Check that you have a complete exam. Your exam consists of one cover sheet, followed by six problems on eight pages.
- Check that your cell phone is OFF and put it away for the duration of the exam.
- There are multiple versions of the exam. It will be apparent if you copy someone else's work. Students found engaging in academic misconduct will receive a 0 on this exam.
- You are allowed to use a calculator, a ruler, and one sheet of handwritten notes.
- We can only give you credit for computations that appear on your exam. Show **all** your work.
- If you use a trial and error method when an algebraic method is available, you will not receive full credit.
- Write your answers in the specified locations.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so. If you still need more paper, please ask for some.
- Raise your hand if you have a question.
- You have 3 hours to complete the exam.

GOOD LUCK!

1. (24 points – 4 points each) Consider four different bank accounts.

**Account A:** annual interest rate of 2%, compounded continuously

**Account B:** annual interest rate of 2.015%, compounded quarterly

**Account C:** annual interest rate of  $(r \times 100)\%$ , compounded semi-annually

**Account D:** annual interest rate of  $(r \times 100)\%$ , compounded continuously

(a) If you deposit \$5175 in account *A*, how much interest will you earn in 9 months?

ANSWER: \_\_\_\_\_dollars

(b) How much must you deposit into account *B* in order to have \$60,000 in 15 years?

ANSWER: \_\_\_\_\_dollars

(c) You deposit  $\$P$  into account *B*. How long does it take to double your principal?

ANSWER: \_\_\_\_\_years

- (d) Bob deposits \$4800 in account  $B$  on the same day that Chuck deposits \$5000 in account  $C$ . In 42 months, Bob's balance is the same as Chuck's. What is Chuck's interest rate?

ANSWER: \_\_\_\_\_%

- (e) You deposit \$7000 in account  $A$  today. What is the percentage change in the balance from  $t = 5$  to  $t = 6.25$  years?

ANSWER: \_\_\_\_\_%

- (f) A balance in account  $D$  grows from \$600 to \$1080 in 10 years. What is the interest rate?

ANSWER: \_\_\_\_\_%

2. (14 points) You manufacture and sell *Things*. The average cost ( $AC$ , in dollars) of manufacturing  $q$  of your Things is given by

$$AC(q) = 23 + \frac{7300}{q}.$$

The total revenue ( $TR$ , in dollars) from selling  $q$  of your Things is

$$TR(q) = 106q - q^2.$$

- (a) (3 points) Write out a formula for total cost ( $TC$ ) as a function of  $q$ . Write it in the form  $TC = mq + b$ .

ANSWER:  $TC =$  \_\_\_\_\_

- (b) (3 points) What is the value of your fixed costs ( $FC$ )?

ANSWER:  $FC =$  \_\_\_\_\_ dollars

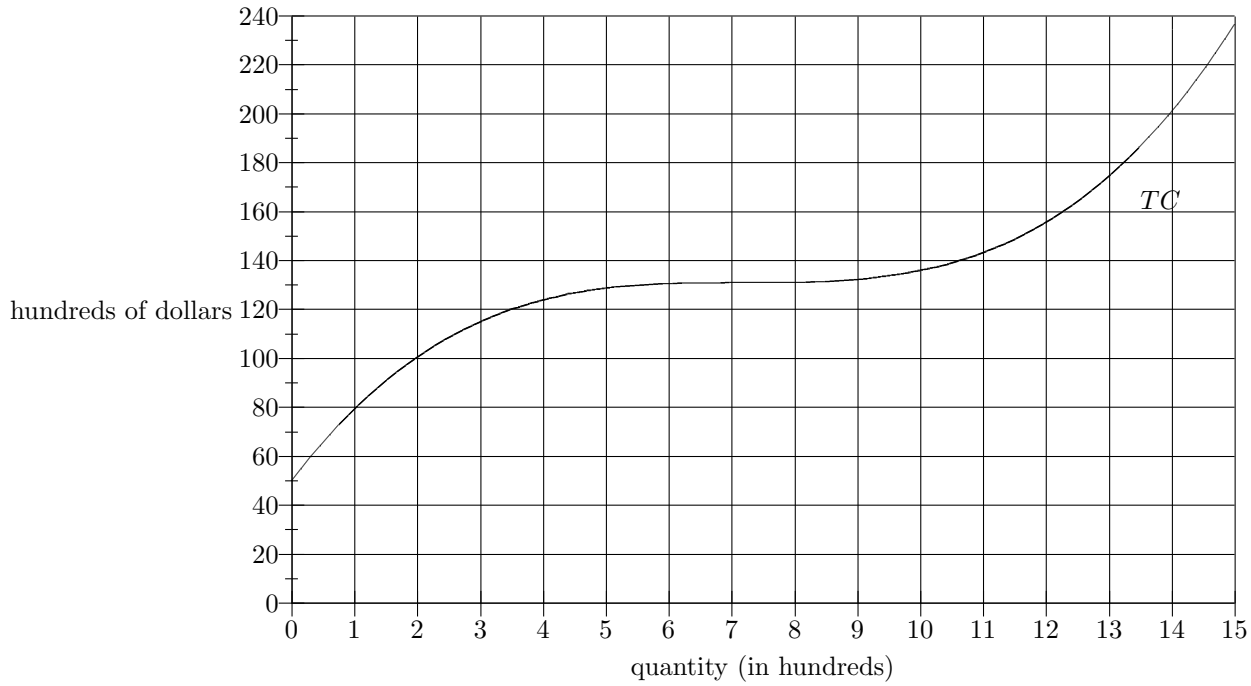
- (c) (3 points) What is the marginal revenue ( $MR$ ) at  $q = 20$  Things?

ANSWER:  $MR(20) =$  \_\_\_\_\_ dollars

- (d) (5 points) Find all positive values of  $q$  at which marginal revenue ( $MR$ ) is equal to marginal cost ( $MC$ ).

ANSWER:  $q =$  \_\_\_\_\_ Things

3. (11 points) You sell *Items*. The graph below is your total cost.



(a) (3 points) Compute your break even price.

ANSWER: \_\_\_\_\_ dollars

(b) (2 points) Give the largest interval over which marginal cost is increasing.

ANSWER: from  $q =$  \_\_\_\_\_ to  $q =$  \_\_\_\_\_ hundred Items

(c) (3 points) Compute the average variable cost of producing two hundred Items.

ANSWER:  $AVC =$  \_\_\_\_\_ dollars per item

(d) (3 points) Your Items sell for \$20 each. What is the smallest quantity you can sell and not be forced to take a loss?

ANSWER:  $q =$  \_\_\_\_\_ hundred Items

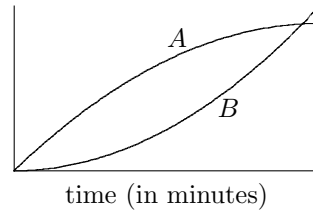
4. (24 points)

Two cars travel along a long, straight road. We have formulas for the distance traveled (in miles) by each car after  $t$  minutes:

$$A(t) = -0.02t^2 + 2.4t$$

$$B(t) = 0.025t^2$$

The distance graphs are given at right.



(a) (4 points) Find the time at which car  $B$ 's distance is 42 miles.

ANSWER:  $t =$  \_\_\_\_\_ minutes

(b) (4 points) Find the time at which car  $A$  is ahead of car  $B$  by the greatest distance.

ANSWER:  $t =$  \_\_\_\_\_ minutes

(c) (4 points) Find the time at which car  $A$ 's average trip speed is 1.27 miles per minute.

ANSWER:  $t =$  \_\_\_\_\_ minutes

(d) (5 points) Find the *three* times at which the cars are 4 miles apart.

ANSWER:  $t =$  \_\_\_\_\_ minutes

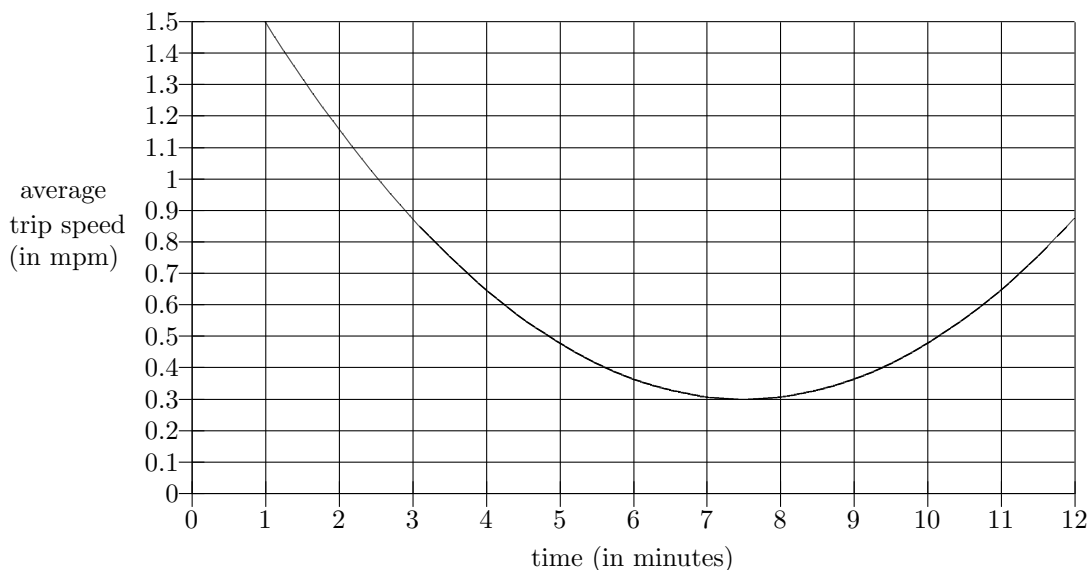
(e) (5 points) Find a formula for car  $A$ 's average speed over a 2-minute interval, starting at time  $t$ .

ANSWER: average speed = \_\_\_\_\_

(f) (2 points) Find a 2-minute interval over which car  $A$ 's average speed is 1.73 miles per minute.

ANSWER: from  $t =$  \_\_\_\_\_ to  $t =$  \_\_\_\_\_ minutes

5. (15 points) At time  $t$  minutes, a car is  $D(t)$  miles from its starting location. The graph below shows the car's **average trip speed** at time  $t$  minutes. That is, the graph shown is of  $\frac{D(t)}{t}$ .



- (a) (2 points) Find a time at which the car's average trip speed is 1.1 mpm.

ANSWER:  $t =$  \_\_\_\_\_ minutes

- (b) (3 points) How far has the car traveled when its average trip speed is at its lowest?

ANSWER: \_\_\_\_\_ miles

- (c) (5 points) How far does the car travel between the two times at which its average trip speed is 0.5 mpm?

ANSWER: \_\_\_\_\_ miles

- (d) (5 points) Compute the average (incremental) speed of the car over the interval from  $t = 3$  to  $t = 11$ .

ANSWER: \_\_\_\_\_ mpm

6. (12 points – 3 points each) You have a bacteria colony whose population at time  $t$  hours is  $p = f(t) = k \cdot m^t$  million bacteria.

(a) The population increases by 44% from time  $t = 6$  hours to  $t = 8$  hours. Find the value of  $m$ .

ANSWER:  $m =$  \_\_\_\_\_

(b) The population at  $t = 5$  hours is 8.2 million bacteria. Find the value of  $k$ . (You'll need to use the answer to (a).)

ANSWER:  $k =$  \_\_\_\_\_

NOTE: You'll need the answers to parts (a) and (b) to do the rest of the problem. If you are unable to complete parts (a) and (b), you may use the values  $m = 2.3$  and  $k = 1.7$  to do parts (c) and (d) for partial credit.

(c) How many bacteria are present at  $t = 12$  hours?

ANSWER: \_\_\_\_\_ million bacteria

(d) At what time will there be 16.5 million bacteria present?

ANSWER:  $t =$  \_\_\_\_\_ hours