MATH 111

Winter 2004 – Final Exam

Hints and Answers

1. (a) HINT: Use the formula $A(t) = Pe^{rt}$ and compute $A\left(\frac{9}{12}\right)$ to get the balance after 9 months. Subtract the principal to obtain the amount of interest.
   ANSWER: \$78.21

   (b) HINT: Solve the equation $60000 = P\left(1 + \frac{0.02015}{4}\right)^{4(15)}$ for P.
   ANSWER: \$44,382.86

   (c) HINT: Set $2P = P\left(1 + \frac{0.02015}{4}\right)^{4t}$ and solve for t.
   ANSWER: $t = 34.49$ years

   (d) HINT: Bob's balance after 42 months ($\frac{42}{12}$ years) is \$5149.83. Set $5149.83 = 5000\left(1 + \frac{r}{2}\right)^{2(\frac{42}{12})}$ and solve for r.
   ANSWER: 0.8454%

   (e) HINT: Use the formula $A(t) = Pe^{rt}$. $A(5) = \$7736.20$ and $A(6.25) = \$7932.04$. This is a change of \$195.84. The percentage change is then \$195.84 divided by \$7736.20.
   ANSWER: 2.53%

   (f) HINT: Solve the equation $1080 = 600e^{10r}$ for r.
   ANSWER: 5.88%

2. (a) HINT: $TC(q) = q \times AC(q)$
   ANSWER: $TC(q) = 23q + 7300$

   (b) HINT: $FC = TC(0)$
   ANSWER: $FC = \$7300$

   (c) HINT: $MR(20) = TR(21) - TR(20)$
   ANSWER: $MR(20) = \$65$

   (d) HINT: $MR(q) = TR(q+1) - TR(q) = 105 - 2q$ (This requires some algebra that I'm not showing you.) Since TC is linear, marginal cost is always the same: the slope of TC. That is, $MC(q) = 23$ for all values of q. Set $105 - 2q$ equal to 23 and solve for q.
   ANSWER: $q = 41$

3. (a) HINT: The breakeven price is the slope of the line that is both a diagonal line and a tangent line.
   ANSWER: approximately \$13

   (b) ANSWER: from $q \approx 7$ to $q = 15$ hundred Items

   (c) HINT: $TC(2) \approx \$100$ and $FC = \$50$. So, $VC(2) \approx \$50$. $AVC(2) = \frac{VC(2)}{2}$.
   ANSWER: $AVC \approx 25$ dollars per item

   (d) HINT: Since items all sell for the same amount, TR is linear. Its slope is the price per item: 20. Draw the graph of TR and find where it intersects TC.
   ANSWER: $q \approx 6.5$ hundred Items

4. (a) HINT: Solve $0.025t^2 = 42$ for t.
   ANSWER: $t = 40.99$

   (b) HINT: Find a formula for $D(t)$, the distance between the cars: $D(t) = -0.045t^2 + 2.4t$. This is a quadratic function whose graph is a parabola that opens downward. It is maximized at its vertex.
   ANSWER: $t = 26.67$ minutes

   (c) HINT: Car A's average trip speed is $\frac{A(t)}{t} = -0.02t + 2.4$. Set this equal to 1.27 and solve for t.
   ANSWER: $t = 56.5$ minutes

(d) HINT: The cars will be four miles apart when $A(t) - B(t) = 4$ AND when $B(t) - A(t) = 4$. This gives two quadratic equations that simplify to $0.045t^2 - 2.4t + 4 = 0$ and $0.045t^2 - 2.4t - 4 = 0$. Use the quadratic formula on each of these equations.

ANSWER: $t = 1.72,\ 51.61,\ 54.95$ minutes

(e) HINT: Compute and simplify the formula for car A's average speed:

$$\frac{A(t+2) - A(t)}{2}.$$

ANSWER: average speed $= -0.04t + 2.36$

(f) HINT: Set $-0.04t + 2.36$ equal to 1.73 and solve for t. That gives the beginning of the two-minute interval.

ANSWER: from $t = 15.75$ to $t = 17.75$ minutes

5. (a) ANSWER: $t \approx 2.2$ minutes

(b) HINT: Average trip speed is lowest at $t = 7.5$ minutes. At that time, the average trip speed is 0.3 mpm. So $\frac{D(7.5)}{7.5} = 0.3$. Solve for $D(7.5)$.

ANSWER: 2.25 miles

(c) HINT: Average trip speed is 0.5 mpm at $t \approx 4.8$ and $t \approx 10.2$. So, $\frac{D(4.8)}{4.8} = 0.5$ and $\frac{D(10.2)}{10.2} = 0.5$. Find $D(10.2) - D(4.8)$.

ANSWER: ~ 2.7 miles

(d) HINT: Using the same method as above, compute $D(3)$ and $D(11)$. Then the average speed is

$$\frac{D(11) - D(3)}{8}.$$

ANSWER: ~ 0.56 mpm

6. (a) HINT: The populations make up a multiplicative sequence with multiplier m. To get from the population at $t = 6$ to the population at $t = 8$, multiply by m^2. That is, $f(8) = m^2 \times f(6)$. But we also know that $f(8) = 1.44 \times f(6)$. So, $m^2 = 1.44$. Solve for m.

ANSWER: $m = 1.2$

(b) HINT: $f(5) = k \times 1.2^5 = 8.2$. Solve for k.

ANSWER: $k = 3.295396$

(c) HINT: $f(t) = 3.295396 \times 1.2^t$. Compute $f(12)$.

ANSWER: $f(12) = 29.38$ million bacteria

(d) HINT: Solve $16.5 = 3.295396 \times 1.2^t$ for t.

ANSWER: $t = 8.84$ hours