MATH 111 — Winter 2005 Final Exam

Hints and Answers

1. (a) HINT: Profit is positive when $TR > TC$: from $q \approx 1.2$ to $q \approx 8.8$. Profit is maximized at $q \approx 4.8$ (where $MR = MC$). Profit increases from $q = 0$ to $q \approx 4.8$.
   ANSWER: from $q \approx 1.2$ to $q \approx 4.8$

   (b) HINT: MC is the same at all quantities: the slope of the TC graph.
   ANSWER: ~ 2

   (c) HINT: Average cost is the slope of the diagonal line through TC.
   ANSWER: ii

   (d) HINT: Profit is maximized at $q \approx 4.8$. Max profit is the vertical distance between TR and TC at this quantity.
   ANSWER: ~ 15

2. (a) ANSWER: $q \approx 70$, 175, 215

   (b) HINT: $AC(450) = 2.75$. So, $TC(450) = 2.75 \cdot 450$.
   ANSWER: ~ 1237.50

   (c) HINT: $AC(200) = 3.75$. So, $TC(200) = 3.75 \cdot 200 = 750$. $FC = TC(200) - VC(200)$.
   ANSWER: ~ 250

   (d) HINT: Breakeven price is the lowest value of AC.
   ANSWER: 1.75

3. (a) ANSWER: $\frac{B(7)}{7} = 1.89$

   (b) ANSWER: $\dfrac{A(5) - A(2)}{5 - 2} = 1.726$

   (c) HINT: Set $A(t) = B(t)$: $3 - \sqrt{t} + 2t = 5\sqrt{t}$. Move all the $\sqrt{t}$'s to one side: $3 + 2t = 6\sqrt{t}$. Square both sides: $9 + 12t + 4t^2 = 36t$. This is now a regular old quadratic equation. Solve the usual way.
   ANSWER: 0.402 and 5.598

4. (a) ANSWER: $TR(q) = pq = 21q - 0.4q^2$; $MR(q) = TR(q+1) - TR(q) = 20.6 - 0.8q$ (simplified)

   (b) HINT: There are two ways:

   - $TC(q) = AC(q) \times q = 12q + 16$. You can now find a formula for profit at q: $P(q) = TR(q) - TC(q)$. It is a quadratic whose graph is a parabola that opens down. So, its max occurs at the vertex. The q-coordinate of the vertex is $q = 11.25$. Since we can't sell 0.25 Objects, we round to the *nearest* whole number.
   - $TC(q) = AC(q) \times q = 12q + 16$. Then, $MC(q) = TC(q+1) - TC(q) = 12$. Set $MR = MC$ and solve for q: $q = 10.75$. Round up to the *next* whole number.

   ANSWER: Either way, the profit is maximized at $q = 11$ Objects.

   (c) HINT: Profit is given by $P(q) = TR(q) - TC(q) = -0.4q^2 + 9q - 16$. Compute $P(11)$.
   ANSWER: 34.6

5. (a) $ATS(t) = at + b$

   (b) HINT: The formula for ATS is linear. Its graph is a line that goes through the points $(5, 35)$ and $(9, 25)$. Find the slope and y-intercept.
   ANSWER: $a = -2.5$ and $b = 47.5$

(c) HINT: Set $ATS(t) = -2.5t + 47.5$ equal to 28.4 and solve for t. Plug that time into $D(t) = -2.5t^2 + 47.5t$.
ANSWER: 216.976

6. (a) ANSWER: Sequence A is multiplicative with multiplier $\frac{1}{4} = 0.25$. A recursive formula would be $A(k+1) = \frac{1}{4}A(k)$. An explicit formula would be $A(k) = 160 \cdot (\frac{1}{4})^k$.

(b) ANSWER: Sequence B is neither additive nor multiplicative.

(c) ANSWER: Sequence C is additive with increment 0.08. A recursive formula would be $C(k+1) = C(k) + 0.08$. An explicit formula would be $C(k) = 5 + 0.08k$.

7. (a) HINT: $1,000,000 = P(1 + \frac{0.0575}{12})^{12(25)}$. Solve for P.
ANSWER: 238.337.64

(b) HINT: $8818 = 8000(1 + 0.09t)$. Solve for t.
ANSWER: 1.14

(c) HINT: $150,000 = P(1 + 0.02)^{30}$. Solve for P.
ANSWER: 82,810.63

(d) HINT: $A_1 = 25,000(1.0125)^5 = \$26,602.05$; $A_2 = 25,000e^{0.06(5)} = \$33,746.47$. Add A_1 and A_2 to get the total value.
ANSWER: 60,348.52

8. (a) ANSWER: A: 8.75%; B: 10.79

(b) HINT: $2P = P(1 + \frac{0.0845}{6})^{6t}$. Solve for t.
ANSWER: 8.26

(c) HINT: $5280 = 3000e^{0.1025t}$. Solve for t.
ANSWER: 5.52

(d) HINT: $12,000 = 4,000(1 + \frac{r}{4})^{4(20)}$. Solve for r. (This does not require logarithms.)
ANSWER: 0.0553

(e) HINT: After 40 months, the amount in account A is \$13,227.35. Solve $13,227.35 = Pe^{0.1025(40/12)}$ for P.
ANSWER: 9,399.16