

MATH 111 - Winter 2006
Final Exam
March 11, 2006

Name _____

Student ID # _____

Section _____

HONOR STATEMENT

“I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam.”

SIGNATURE: _____

1	24	
2	15	
3	14	
4	17	
5	16	
6	14	
Total	100	

- Check that your exam contains eight pages (including this cover sheet).
- Please turn your cell phone OFF and put it away for the duration of the exam.
- Unless otherwise indicated, you must show your work. The correct answer with no supporting work may result in no credit.
- On problems that require you to work with a graph, show your work by clearly marking all lines and points that you use.
- If you use a guess-and-check method when an algebraic method is available, you may not receive full credit.

GOOD LUCK!

1. (24 points)

- (a) A bacteria colony quadruples its population every 50 minutes. If there are 100 bacteria in the colony now, how many will be in the colony 2.5 hours from now?

ANSWER: _____ bacteria

- (b) How much must you deposit in an account paying 3% annually, compounded quarterly, to have \$734 in 10 months?

ANSWER: \$ _____

- (c) You have \$1200 to deposit into an account in which interest is compounded continuously. What is the smallest annual interest rate the account can have if you are to have \$1800 in the account in 7 years?

ANSWER: _____%

- (d) You deposit \$700 in an account that pays 1.33% per year, compounded annually. What is the percentage change in the balance after five months?

ANSWER: _____%

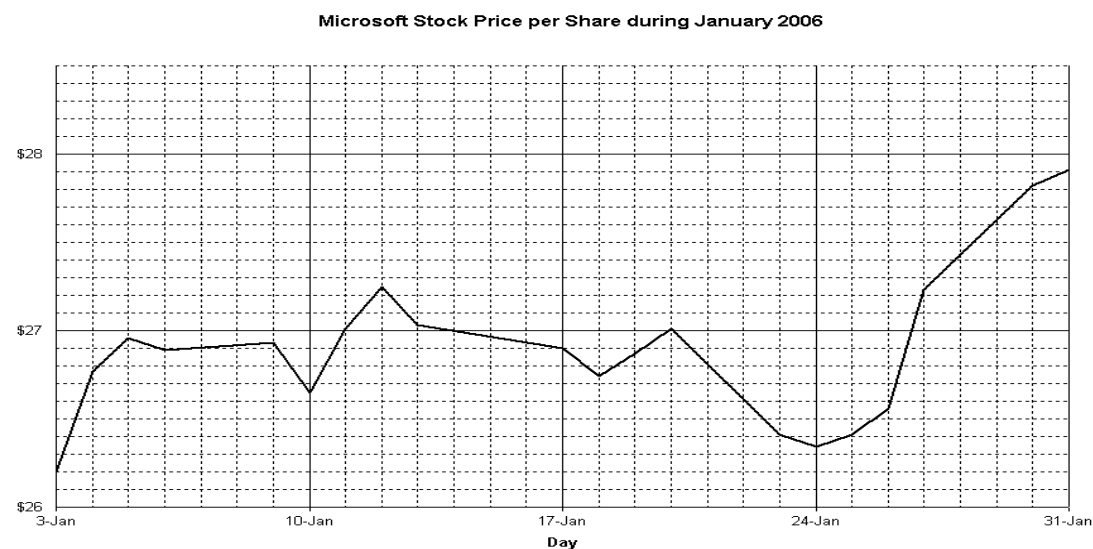
- (e) Money is compounded continuously at a rate of 11.2% per year. What is the present value of \$1 four years from now?

ANSWER: \$ _____

- (f) How long does it take to triple your principal in an account paying 8.75% per year, compounded monthly?

ANSWER: _____years

2. (15 points) The following graph shows the price for a share of MSFT stock during the month of January 2006.



Let $P(t)$ denote the price of one share of MSFT stock on the t^{th} day of the month. That is, the price on January 3rd is denoted by $P(3)$.

- (a) i. Translate into functional notation: The change in the stock price was greater during the first week (from 3-Jan to 10-Jan) than during the second week (from 10-Jan to 17-Jan).
- ii. Is the statement in (i) true or false? (Carefully use the graph to determine the answer and clearly indicate how you got your answer from the graph.)

ANSWER: (circle one) True False

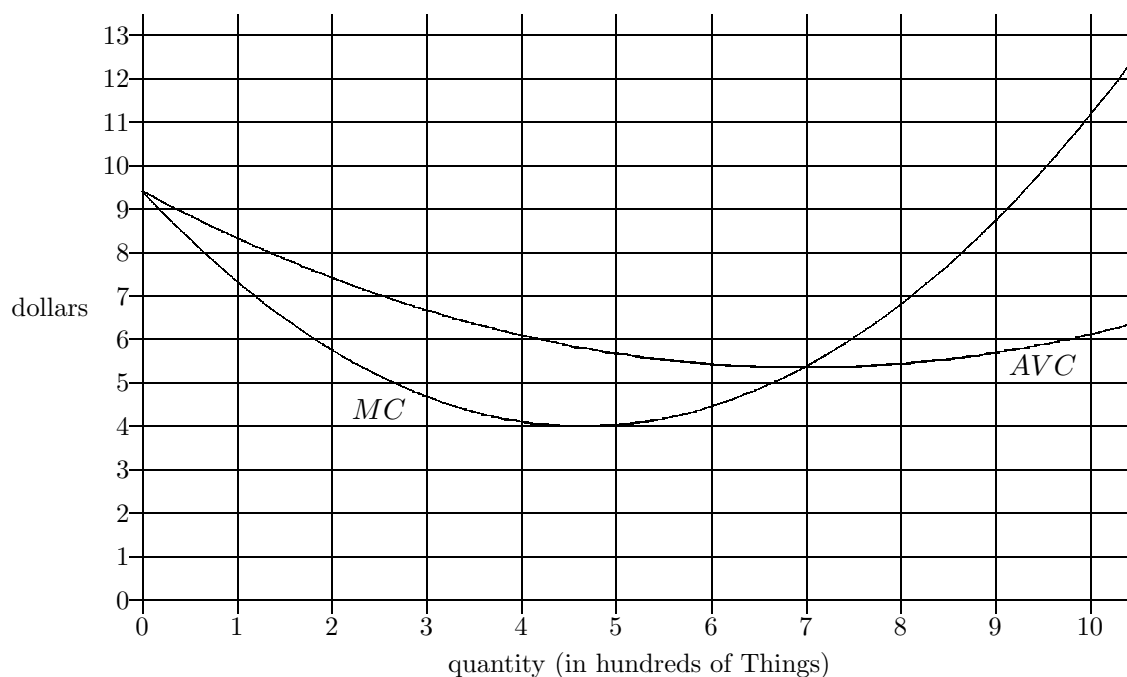
- (b) i. Translate into functional notation: The (overall) average rate of change of the stock price during the first five trading days (from 3-Jan to 8-Jan) was greater than the overall average rate of change during the entire month (from 3-Jan to 31-Jan).
- ii. Is the statement in (i) true or false? (Carefully use the graph to determine the answer and clearly indicate how you got your answer from the graph.)

ANSWER: (circle one) True False

- (c) On what day of January was the overall average rate of change of the stock price \$0.05 per day?

ANSWER: January _____

3. (14 points) You make and sell Things. The following graph shows the marginal cost (MC) and average variable cost (AVC) of manufacturing Things.



- (a) What is the shutdown price for selling Things?

ANSWER: \$ _____

- (b) What is the change in total cost (TC) if production is increased from 600 to 601 Things?

ANSWER: \$ _____

- (c) What is the variable cost (VC) to produce $q = 2$ hundred Things?

ANSWER: _____ hundred dollars

- (d) If you sell Things for \$10 each (that is, the market price is \$10), what quantity will maximize profit?

ANSWER: $q =$ _____ hundred Things

4. (17 points) You sell video game consoles. Your total revenue (in dollars) for selling a batch of q consoles is:

$$TR(q) = -0.2q^2 + 400q.$$

Your marginal cost is given by the formula:

$$MC(q) = -0.2q + 199.9.$$

- (a) Recall that $MR(q) = TR(q+1) - TR(q)$. Use this definition to find the formula for your marginal revenue, $MR(q)$. Simplify it as far as possible.

ANSWER: $MR(q) =$ _____

- (b) What is the maximum value of total revenue?

ANSWER: \$ _____

- (c) It is known that if the marginal cost (in dollars) is given by a formula of the form $MC(q) = Aq + B$, then the formula for the variable cost function (in dollars) is $VC(q) = \frac{A}{2}q^2 + (B - \frac{A}{2})q$. Use this information and the MC formula at the top of the page to determine the formula for $VC(q)$.

ANSWER: $VC(q) =$ _____

- (d) If your fixed costs are $FC = \$20,000$, what is the smallest quantity at which you break even? (Your answer need not be a whole number of consoles.)

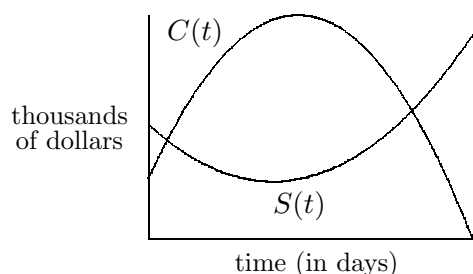
ANSWER: $q =$ _____ consoles

5. (16 points)

You own a small business which has two bank accounts: a checking account and a savings account. The balances of these two accounts (in thousands of dollars) after t days are given by:

Checking : $C(t) = -t^2 + 12t + 13$ and

Savings : $S(t) = 0.5t^2 - 5t + 25$.



(a) After how many days will you run out of money in your checking account?

ANSWER: $t =$ _____ days

(b) Let $A(t)$ be the total amount of money you have in both accounts combined. When will $A(t)$ be largest?

ANSWER: $t =$ _____ days

(c) What is the average rate of change in the balance of your savings account from day 2 to day 10?

ANSWER: _____ thousand dollars per day

(d) You open another store and another checking account whose balance after t days is denoted $N(t)$. The balance of this new account is always the same as the amount that was in the **original checking account** ten days before. Find a formula for $N(t)$. Simplify until your formula is in the form $N(t) = at^2 + bt + c$.

ANSWER: $N(t) =$ _____

6. (14 points) You are the lucky winner of the state lottery drawing. After all taxes have been paid, you can collect your prize in one of two ways:

Option I: 25 payments of \$2.9 million each. You receive the first payment immediately and then one payment each year for the next 24 years. If you choose this option, you will spend all your winnings as you receive them.

Option II: A one-time lump sum of \$41 million, paid immediately. If you choose this option, you will immediately spend a portion of your winnings and invest the rest in an account paying 5% per year, compounded annually.

- (a) What is your total income from Option I?

ANSWER: _____million dollars

- (b) Suppose you select Option II, immediately spend half your winnings, deposit the other half in the account, and withdraw the entire balance at the end of 24 years. Including the money you spent and the money from the account, what is your total income from this scenario?

ANSWER: _____million dollars

- (c) Suppose you select Option II, immediately spend Y million dollars, deposit the rest ($41 - Y$ million dollars) in the account, and withdraw the entire balance at the end of 24 years. What value of Y will cause this scenario to yield the same total income (including the money spent and the money from the account) as the total income from Option I?

ANSWER: $Y =$ _____million dollars