Math 111 - Winter 2008
Final Exam
March 15, 2008

Name: _____

Quiz Section: _____

Student ID Number: _____

| | | |
|---|---|---|
| 1 | 16 | |
| 2 | 12 | |
| 3 | 16 | |
| 4 | 15 | |
| 5 | 12 | |
| 6 | 15 | |
| 7 | 14 | |
| Total | 100 | |

- You are allowed to use a calculator, a ruler, and one **hand-written** 8.5 by 11 inch page of notes.
- Check that your exam contains all the problems listed above.
- You must **show and explain your work** on all problems. The correct answer with no supporting work may result in no credit.
- Unless otherwise stated, round your final answers to two digits after the decimal point.
- Any student found engaging in academic misconduct will receive a score of 0 on this exam. All suspicious behavior will be reported to the student misconduct board. In such an instance, you will be force to meet in front of a board of professors to explain your actions. The board typically decides to either put a student on academic probation or to expel the student.
  DO NOT CHEAT OR DO ANYTHING THAT LOOKS SUSPICIOUS!
  WE WILL REPORT YOU AND YOU MAY BE EXPELLED!
- There are multiple versions of the exam so if you copy off a neighbor and put down the answers from another version we will know you cheated.

GOOD LUCK!

1. (16 points)

(a) (4 pts) How much must you deposit into an account paying 6% annually, compounded quarterly, to have $1500 in 3 years?

ANSWER: _____ dollars

(b) (4 pts) A bacteria colony triples its population every 15 minutes. If there are 10 bacteria now, how many will there be in 2 hours?

ANSWER: _____ bacteria

(c) (4 pts) Find the Annual Percentage Yield (APY) for an account that pays 6.2% annually, compounded monthly.

ANSWER: _____ percent

(d) (4 pts) You deposit $500 into an account that pays 3% per year, compounded continuously. What is the percentage change over the first 1.5 years?

ANSWER: _____ percent

2. (12 points)

(a) (4 pts) Consider the sequences:

$$A(0) = 10, \quad A(1) = 25, \quad A(2) = 62.5, \quad A(3) = 156.25, \quad \ldots$$
$$B(0) = 2018, \quad B(1) = 2070.5, \quad B(2) = 2123, \quad B(3) = 2175.5, \quad \ldots$$

Give the explicit formulas for the sequences $A$ and $B$.

ANSWER: $A(k) =$ _____

$B(k) =$ _____

(b) (4 pts) Let $C(t) = 30m^t$ be the function for the size of a bacteria populations after $t$ hours. If there are 225 bacteria in 3.1 hours, then what is the value of $m$?

ANSWER: $m =$ _____

(c) (4 pts) Let $D(t) = 50(2)^t$ be the function for the size of a different bacteria populations after $t$ hours. After how many hours will there be 1000 bacteria?

ANSWER: $t =$ _____

3. (16 points)

(a) (4 pts) You deposit \$300 into an account that pays 6% interest per year, compounded monthly. What is the value of the account in 5 years and 3 months?

ANSWER: _____ dollars

(b) (4 pts) A bacteria population doubles every 10-minutes. If the population is 848 million bacteria in 3 hours, what is the population in 2 hours and 30 minutes?

ANSWER: _____ million bacteria

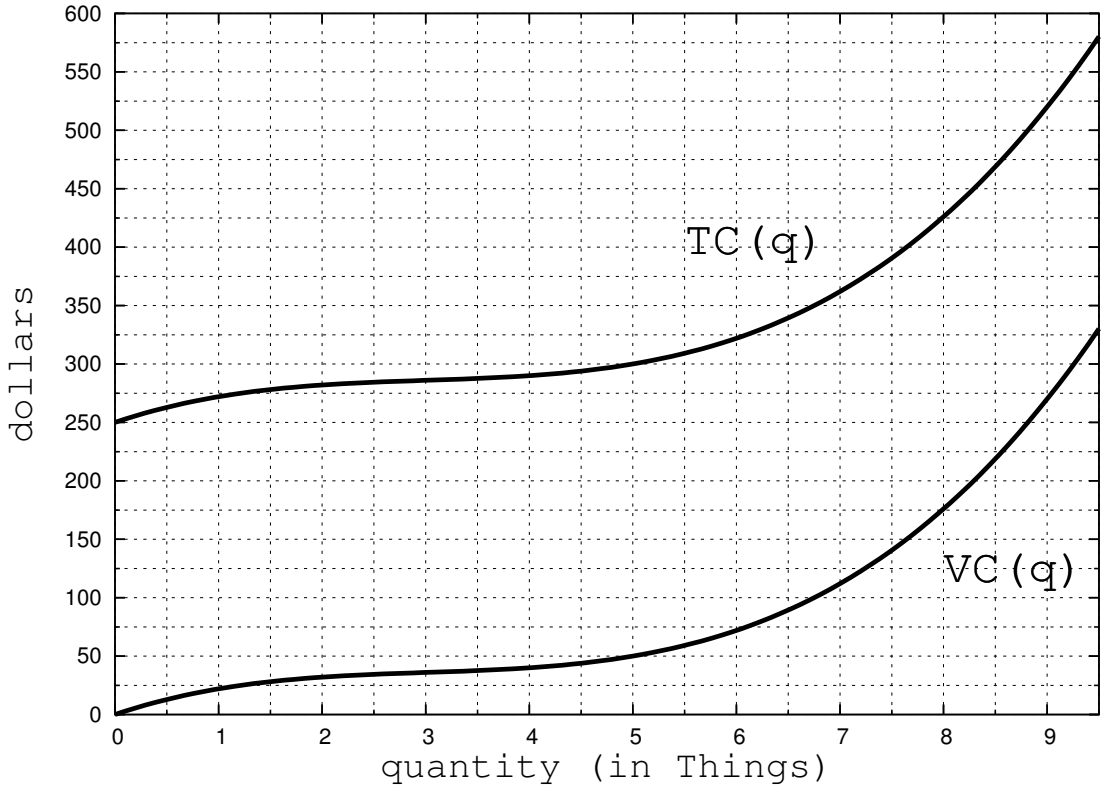
(c) (4 pts) An account pays $(r \times 100)\%$ interest per year, compounded quarterly. The principal is \$1000 and the value in 2 years is \$1400. What is the value of $r$? (Round to 3 digits after the decimal.)

ANSWER: $r =$ _____

(d) (4 pts) How long does it take \$100 to double in an account paying 8% per year, compounded quarterly?

ANSWER: _____ years

4. (15 points) The graphs of Total Cost ($TC$) and Variable Cost ($VC$) for selling Things is given.




**For each of the problems below, explain your work in words and clearly label your work in the graph!**

(a) (3 pts) Compute the average cost of producing 3 Things.

ANSWER: $AC(3) =$ _____ dollars per Thing

(b) (4 pts) Compute the value of marginal cost at $q = 8$ Things.

ANSWER: $MC(8) =$ _____ dollars

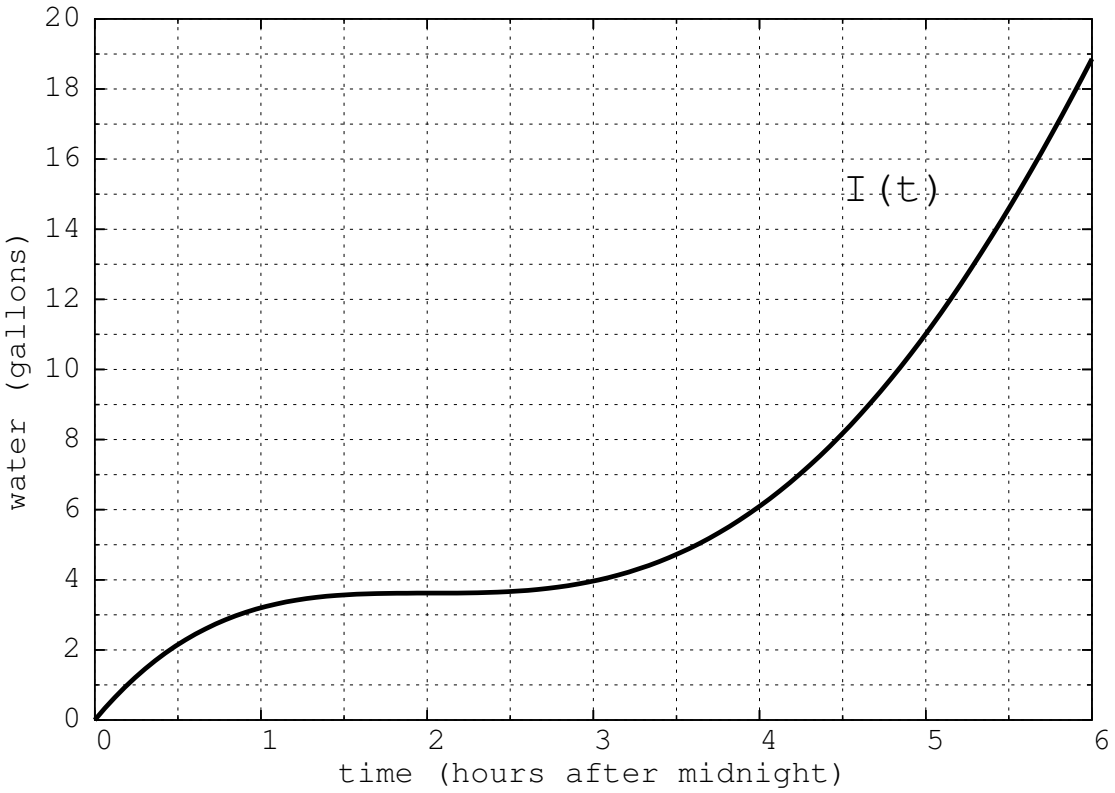
(c) (4 pts) Find the Shutdown Price ($SDP$).

ANSWER: _____ dollars per Thing

(d) (4 pts) The market price per Thing is $p = 60$ dollars. Sketch the straight line graph for Total Revenue and find the first quantity at which profit is zero.

ANSWER: $q =$ _____ Things

5. (12 points) The graph of the flow of water into a reservoir, $I(t)$, is given below. The time $t$ is in hours after midnight and the amount of water is in gallons.




**For each of the problems below, explain your work in words and clearly label your work in the graph!**

(a) (3 pts) Find the average (incremental) rate of flow into the reservoir over the 3 hour interval starting at $t = 3$.

ANSWER: _____ gallons per hour

(b) (3 pts) Find all values of $t$ at which $\dfrac{I(t+2) - I(t)}{2} = 3.5$.

ANSWER: $t =$ _____

(c) (3 pts) Find the lowest overall rate of flow into the reservoir.

ANSWER: _____ gallons per hour

(d) (3 pts) If water flows out at a constant rate of 2 gallons per hour, then what is the largest shortage of water that would occur during the 6 hours after midnight?

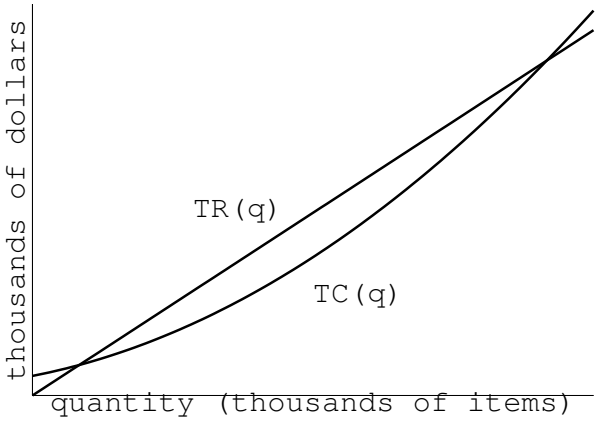
ANSWER: _____ gallons

6. (15 points)

The graphs of Total Revenue and Total Cost for selling **thousands** of Items are given.
The formula for Total Cost is

$$TC(q) = q^2 + 6q + 9 \text{ (**thousand** dollars)},$$

where $q$ is in **thousands** of Items.
Total Revenue is a **linear** function that goes through the points (0,0) and (2,40).




(a) (3 pts) Find the formula for Total Revenue ($TR$).

ANSWER: $TR(q) =$ _____

(b) (4 pts) Find all values of $q$ at which the average variable cost ($AVC$) is equal to 8 dollars per Item?

ANSWER: $q =$ _____ thousand Items

(c) (4 pts) What is the smallest value of $q$ you can sell and not lose any money?

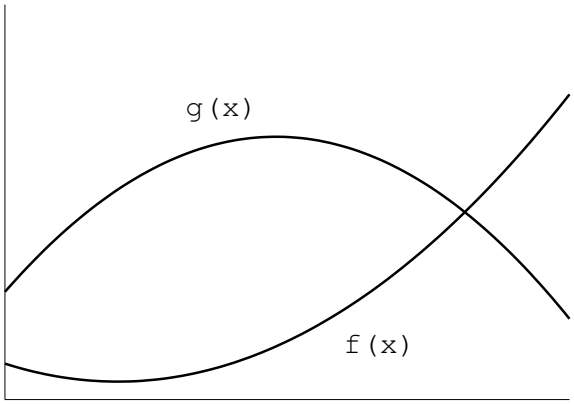
ANSWER: $q =$ _____ thousand Items

(d) (4 pts) At what quantity is profit greatest? (Round your quantity to the nearest **Item**).

ANSWER: _____ **Items**

7. (14 points)

The graphs of the quadratic functions $f(x)$ and $g(x)$ are given.




The formula for $f(x)$ is

$$f(x) = 2x^2 - 10x + 25.$$

The formula for $g(x)$ is of the form

$$g(x) = -3x^2 + bx + 75,$$

where $b$ is a number.

**The vertex for $g(x)$ occurs at $x = 6$.**

(a) (4 pts) Evaluate $\dfrac{f(3) - f(0)}{3}$

ANSWER: $\frac{f(3)-f(0)}{3} =$ _____

(b) (4 pts) Find the longest interval when $f(x)$ and $g(x)$ are both increasing.

ANSWER: from $x =$ _____ to $x =$ _____

(c) (2 pts) Find the number $b$.

ANSWER: $b =$ _____

(d) (4 pts) Write out and simplify the formula for $f(x + 3) - f(x)$.

ANSWER: $f(x + 3) - f(x) =$ _____