

NAME: _____

QUIZ SECTION: _____

Student ID #: _____

**Math 111 -- Winter 2011
Final Exam**

HONOR STATEMENT:

“I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam.”

SIGNATURE: _____

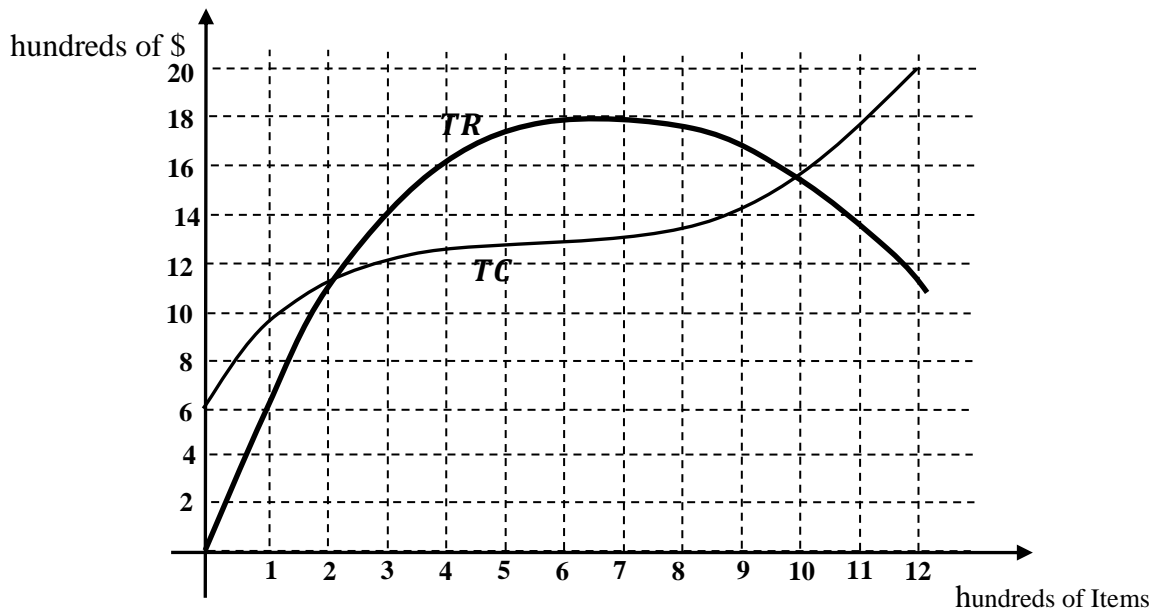
INSTRUCTIONS:

- When the exam starts, verify that your exam contains **8 pages** (including this cover page).
- Please turn your cell phone OFF and put it away for the duration of the exam.
- Unless specifically instructed otherwise, you **must show all your work in order to get full credit**. The correct answer with incorrect or missing work may result in little or no credit.
- On problems in which you use a graph, show your work by clearly drawing & labeling any lines and points you use.
- If you use a graphing calculator or a guess-and-check method when an algebraic method is available, you will not receive full credit.
- **You may round your final answers to two decimal digits. Don't round any values prior to the final answer.**
- You are allowed to use a calculator, a ruler, and one sheet of notes. You have 3 hours for this exam.

GOOD LUCK!

Problem 1	12	
Problem 2	10	
Problem 3	10	
Problem 4	6	
Problem 5	16	
Problem 6	18	
Problem 7	16	
Problem 8	12	
Total:	100	

1 (12 pts) The following are the Total Revenue and Total Costs graphs for producing and selling Items.



Use the methods studied in this class to answer the following questions.

Draw and label on the graph any lines you use, show your calculations, and be as precise as possible.

a) Estimate the profit from producing and selling 7 hundred Items.

Answer: Profit(7) = _____ hundred Items

b) Estimate the price per item p for an order of 3 hundred items.

Answer: $p(3)$ = _____ \$/Item

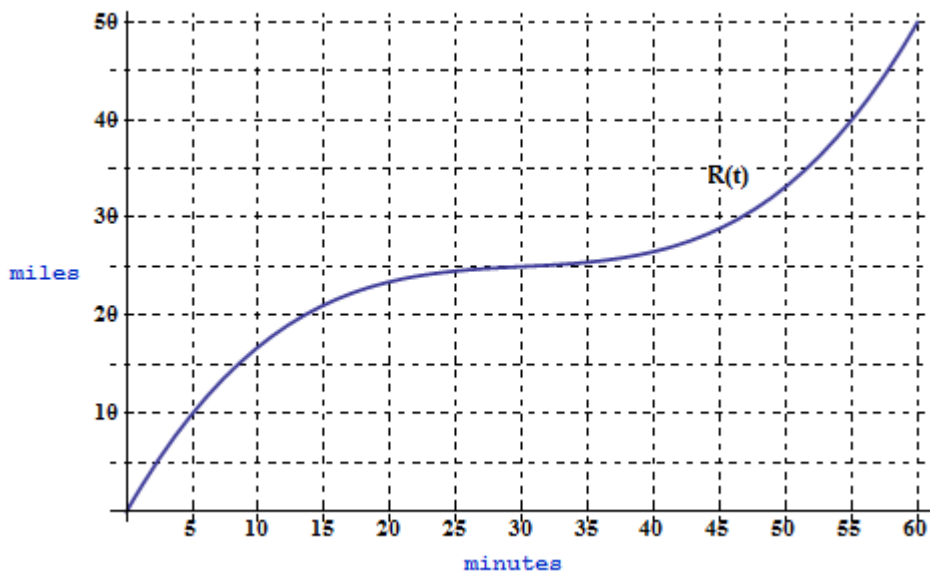
c) Compute the average variable cost (AVC) of producing 6 hundred Items.

Answer: $AVC(6)$ = _____ \$/Item.

d) Find the longest range of quantities q at which the **marginal cost** MC is less than 2 dollars per Item

Answer: From q = _____ to q = _____ hundred items

2 (10 pts) A Red car travels along a long, straight road. Information about the Red car's distance, $R(t)$, at various times is given in the graph below. Distances are in miles, and times are in minutes.



Use the methods studied in this class to answer the following questions. Draw and label on the graph any lines you use, show your calculations, justify your answers, and be as precise as possible.

- a) Find the longest time interval during which the Red car's average trip speed decreased while its average speed over 5 minute intervals increased.

Answer: from $t =$ _____ to $t =$ _____ minutes

- b) Find a time t such that $\frac{R(t+10)-R(t)}{10} = 0.25$ (0.5)

Answer: $t =$ _____ minutes

- c) A Yellow car starts from the same place as the Red car, but 10 minutes later. For every place along the highway, the Yellow car reaches that place exactly 10 minutes after the Red car did. Let $Y(t)$ denote the distance traveled by the Yellow car.

- i. Estimate $Y(15)$

Answer: $Y(15) =$ _____ miles

- ii. Circle all the correct translations of the following functional notation statement:

$$R(t) = Y(t) + 10 \text{ for all values of } t,$$

regardless of whether this statement is true or not. No need to show work or justify your choices.

- The graph of $Y(t)$ is the same as the graph of $R(t)$ shifted to the left 10 horizontal units.
- The graph of $Y(t)$ is the same as the graph of $R(t)$ shifted down 10 vertical units.
- The Red car is always 10 miles ahead of the Yellow car.
- The Yellow car is always 10 miles ahead of the Red car.

3 (10 pts) You sell Things. Your Total Cost, in dollars, for producing q Things is given by the formula:

$$TC(q) = \sqrt{3q^2 + 10,000}$$

a) (2 pts) What is your Fixed Cost?

ANSWER: $FC = \$$ _____

b) (3 pts) What is your Marginal Cost at 50 Things? (HINT: you do not need to find a general formula for MC)

ANSWER: $MC(50) = \$$ _____

c) (5 pts) At what value of q is the **average cost** $AC(q)$ equal to \$2.50? Round your answer to 2 decimal digits.

Answer: at $q =$ _____ Things

4 (6 pts) The graphs below represent the **marginal cost (MC)**, **marginal revenue (MR)**, and the **average cost (AC)** for a business producing and selling lunch boxes. Answer the following questions. No need to show work.

a) What number of lunch boxes produced and sold maximizes your profit?

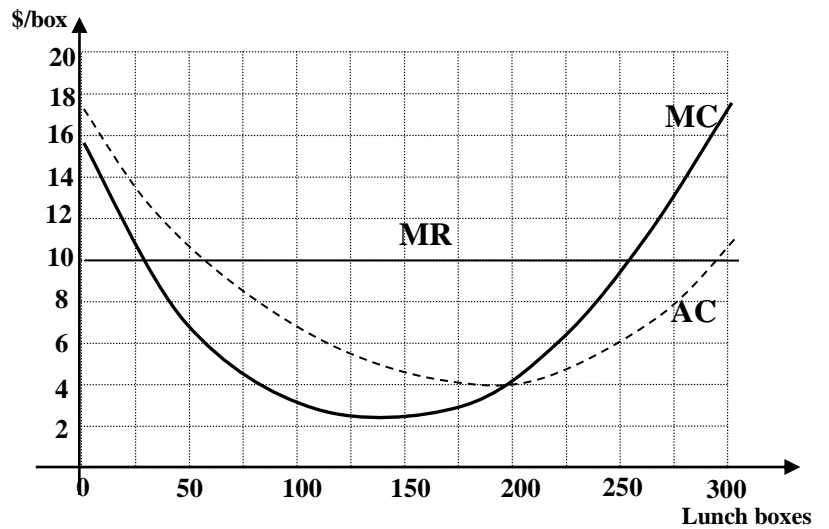
Answer: _____ boxes

b) What is the breakeven price?

Answer: _____ \$/box

c) Estimate your change in profit if you produce and sell 101 boxes instead of 100 boxes.

Profit increases/decreases (circle one) by \$ _____



5 (16 pts) Two functions vary with time t :

$$f(t) = t^2 - 10t + 24$$

$$g(t) = -t^2 + 4t$$

- a) Compute a formula in terms of t for $g(t + 2)$. Simplify as far as possible.

Answer: $g(t + 2) =$ _____

- b) What is the longest time interval, if any, over which both functions are increasing? Justify your answer.

Answer: from $t =$ ____ to $t =$ ____ OR “none exists”

- c) Find all the times when the two graphs cross.

Answer: At $t =$ _____ (list all)

- d) Find the maximum value of the difference $g(t) - f(t)$.

Answer: Maximum value of $g(t) - f(t)$ is: _____

6 (18 pts)

a) (5 pts) Determine whether the following sequence is additive, multiplicative, or neither. If it's additive or multiplicative, compute its increment or multiplier, respectively, and its 100th term. Show work.

A: 3.57, 4.19, 4.81, 5.43,...

Answer: Sequence is Additive with increment _____ and $A(100) =$ _____
Multiplicative with multiplier _____ and $A(100) =$ _____
Neither

b) (8 pts) A bacteria colony has 5 million bacteria at time $t = 0$ minutes, and it doubles every 10 minutes.

i. What is the proportionate change in the bacteria population from 12 to 20 minutes?

Answer: _____ bacteria

ii. At what time will your bacteria population reach 7 million bacteria?

Answer: At $t =$ _____ minutes

c) (5pts) You have \$5,000 to invest in a college-savings account for your child. You wish to have a balance of at least \$20,000 fifteen years from now. What is the minimum interest rate per year, $r \times 100\%$, compounded continuously, that would allow you to achieve the desired balance on your deposit?

Answer: $r \times 100\% =$ _____

7 (16 pts) You have a choice of 3 bank accounts:

Account A: pays 5%, compounded quarterly

Account B: pays 4.9%, compounded monthly

Account C: pays 4.87%, compounded continuously

- a) What is the future value of \$500 placed in account B, 8 years from now?

Answer: \$ _____

- b) You deposit \$10,000 in Account C today. How much should you deposit in Account A today, so you'll have equal balances in both accounts 3 years from today?

Answer: \$ _____

- c) You deposit \$10,000 in Account B. How long until your balance reaches \$25,000?

Answer: _____ years

- d) Which of the three accounts has the highest APY? What is the value of the highest APY?

Answer: Account _____ has the highest APY, namely _____ %

8 (12 pts) You are the lucky winner of the Washington State Lotto Drawing! The jackpot is \$1 million!! According to www.walottery.com, your cash options for collecting your winnings are:

HOW DO YOU WANT YOUR MONEY?					
"50% CASH OPTION": ONE LUMP SUM PAYMENT			"YEARLY OPTION": 25 ANNUAL PAYMENTS		
PRIZE	Cash Payment Before Taxes	Cash Payment After Federal Income Tax Withholding*	PRIZE	Annual Payment Before Taxes	Annual Payment After Federal Income Tax Withholding*
\$1,000,000	\$500,000	\$375,000	\$1,000,000	\$40,000	\$30,000

More precisely, you are considering the following two ways to collect your winnings:

- a) **Option A:** One lump sum payment of \$500,000. After taxes, you collect \$375,000 in winnings. From the amount you collect, you plan to spend \$75,000 immediately, and to invest the remaining \$300,000 into an account paying 3.75% per year, compounded monthly. Compute how much money, in total, you will receive over 24 years (including the balance in your bank account after 24 years, and the money you spent upfront)

Answer: \$ _____

- b) **Option B:** 25 annual payments of \$40,000. After taxes, you collect 25 annual payments of \$30,000 (one now and one for each of the next 24 years). Compute how much money you will receive if you take Option B.

Answer: \$ _____

- c) Suppose you chose Option A, and invested \$300,000 in the described bank account, but you made one withdrawal of \$50,000 from your bank account 18 **months** after opening the account. What is your bank account balance 24 years after opening the account?

Answer: \$ _____