

MATH 112 - Spring 2004
Exam 1, Version 1 - Hints and Answers

1. (4 points each)

(a) ANSWER: $f'(x) = 6x^2 - 60x + 96$, $g'(x) = 2x - 6.8$

(b) ANSWER: $2a + h - 6.8$

(c) HINT: Find where each function has a horizontal tangent line by solving $g'(x) = 0$ and $f'(x) = 0$. $g(x)$ is increasing from $x = 3.4$ on and $f(x)$ is decreasing from $x = 2$ to $x = 8$.

ANSWER: from $x = 3.4$ to $x = 8$

(d) HINT: $f'(x)$ is a quadratic, whose graph is a parabola that opens upward. It is smallest at its vertex.

ANSWER: $x = 5$

2. (a) (2 points) HINT: Take $q_1 = 2$ and $q_2 = 4$ and compute $P(4) - P(2)$.

ANSWER: 1.875 thousand dollars

(b) (4 points) HINT: Take $q_1 = 5$ and $q_2 = 7$. Compute $P(7) - P(5)$. Substitute 8.7 in for $P(7)$ and solve for $P(5)$.

ANSWER: $P(5) = 7.95$

(c) (4 points) HINT: Take $q_1 = 3$ and $q_2 = 8$. Compute $P(8) - P(3)$ and divide it by 5.

ANSWER: $\frac{P(8) - P(3)}{5} = 0.46875$

(d) (4 points) HINT: Take $q_1 = q$ and $q_2 = q + h$. Compute $P(q + h) - P(q)$ and divide it by h .

ANSWER: $\frac{P(q + h) - P(q)}{h} = \frac{3}{32}(16 - 2q - h)$

(e) (4 points) HINT: In your answer to part (d), let h go to 0 to get $P'(q)$. Set $P'(q) = 1.2$ and solve for q .

ANSWER: $q = 1.6$

3. (a) (6 points)

i. HINT: At $t = 0$, the balloons are at the same altitude. From $t = 0$ to $t = 4$, Laverne's rate of ascent is *positive*, so Laverne's balloon is rising. Shirley's rate of ascent is *negative*, so Shirley's balloon is falling. So, during this interval, the balloons are getting further apart.

ANSWER: $t = 4$

ii. HINT: Both balloons are rising when both rate graphs are positive.

ANSWER: Any four-hour interval between $t = 4$ and $t = 9$.

iii. HINT: Laverne's balloon rises from $t = 0$ to $t = 9$ and then begins to fall.
ANSWER: $t = 9$

(b) (4 points) ANSWER: Laverne: $y = -\frac{4}{3}t + 12$; Shirley: $y = 2t - 8$

(c) (4 points) HINT: The distance between the balloons is greatest when the rate graphs intersect.

ANSWER: $t = 6$ hours

(d) (4 points) HINT: $S'(t)$ is Shirley's rate of ascent. We found in part (b) that $S'(t) = 2t - 8$.

ANSWER: $S'(10) = 12$