NAME: ______________________________ Student ID #: ________________

QUIZ SECTION:______

Math 112 B
Midterm I
April 25, 2006

Problem 1	15	
Problem 2	15	
Problem 3	20	
Total:	**50**	

- You are allowed to use a calculator, a ruler, and one sheet of notes.
- Your exam should contain 4 pages in total and 3 problems.
  Make sure you have a complete test.
- Unless otherwise noted, you **must show how you get your answers**.
  Correct (or incorrect) answers with no supporting work may result in little or no credit.
- If an algebraic method is available, answers obtained by guessing, approximating, or plug-and-check will get little or no credit.
- Write your **final answer in the indicated spaces**. Unless otherwise noted, round your answer to two decimal digits.
- If you need more room, use the backs of pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

GOOD LUCK!

Do you want me to post your grade so far on the class website under the last 4 digits of your STUDENT ID (in about a week)?

☐ *Yes, please post my grade. Sign to give permission:* ______________________________

☐ *No, please don't post my grade so far.*

1) (15 points)

a) (6 points) Compute the derivative *f'(x)* of the function $f(x) = x\left(x^2 + \frac{2}{x^2}\right)$

Rewrite the function in terms of powers of x: $f(x) = x\left(x^2 + \frac{2}{x^2}\right) = x^3 + 2x^{-1}$

Then use the differentiation rules: $f'(x) = 3x^2 - 2x^{-2} = 3x^2 - \frac{2}{x^2}$

Answer: $f'(x) = 3x^2 - 2x^{-2} = 3x^2 - \frac{2}{x^2}$

b) (5 points) Compute the derivative $\frac{dy}{dt}$ of the function $y = \frac{2}{\sqrt{t}} - \sqrt[3]{t} + 0.452$

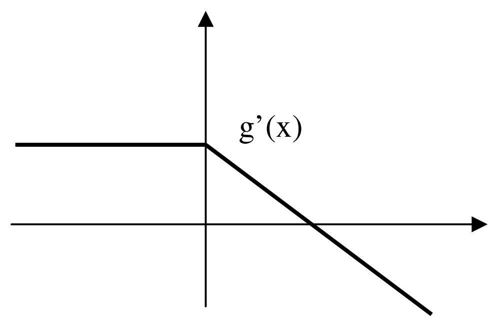
Rewrite the function in terms of powers of t: $y = 2t^{-1/2} - t^{1/3} + 0.452$

Then use the differentiation rules: $\frac{dy}{dt} = 2(-\frac{1}{2}t^{-3/2}) - \frac{1}{3}t^{-2/3} + 0$

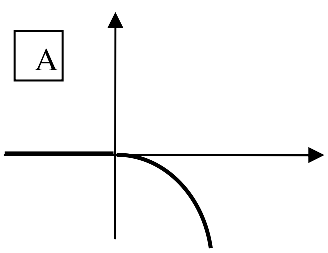
Answer: $\frac{dy}{dt} = -t^{-3/2} - \frac{1}{3}t^{-2/3} = -\frac{1}{\sqrt{t^3}} - \frac{1}{3\sqrt[3]{t^2}}$

c) (4 points) To the right you are given the <u>derived</u> graph *g'(x)* of a function *g(x)*. Which of the graphs labeled A through D below is the graph of the <u>original</u> function *g(x)*?

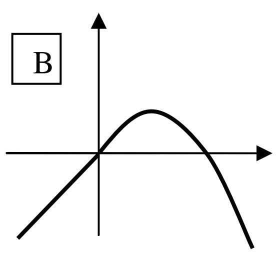
(No need to justify your answer)



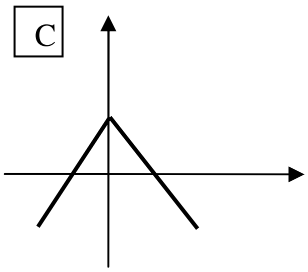
A



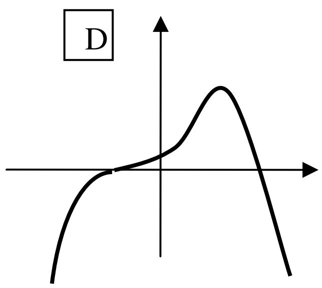
B



C



D

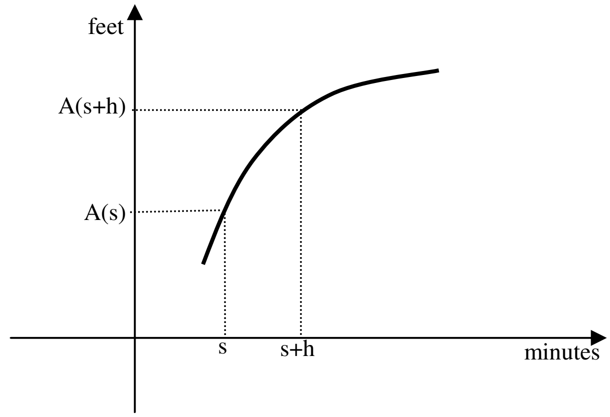


Answer: The graph labeled *g'(x)* above is the derived graph of graph___B______ .

Reason: The given derived graph should come from a function with a constant positive slope to the left of the y-axis (that is, an line which goes up), and with a graph that is first increasing, then decreasing on the right (since the derived graph is first positive, then negative).

2) (15 Points) To the right is a rough sketch of a portion of the graph of the altitude *A(t)* above ground for a weather balloon. The <u>change in the altitude</u> of the balloon, in feet, from time $t=s$ to time $t=s+h$ is given by the formula:

$$A(s+h) - A(s) = -2sh + h^2 + 8h.$$



a) What is the average rate of change of the balloon's altitude from time $t=1$ minute to $t=2.5$ minutes?

Setting s=1, s+h=2.5 (so h=1.5) in the above formula allows us to compute the change in altitude between these two times. So, the average RATE of change is:

$$\frac{\Delta A}{\Delta t} = \frac{A(2.5) - A(1)}{1.5} = \frac{-2(1)(1.5) + (1.5)^2 + 8(1.5)}{1.5} = -2 + 1.5 + 8 = 7.5$$

Answer: 7.5 feet per minute.

b) What is the instantaneous rate of change of the balloon's altitude at time $t=2$ minutes?

First compute the average rate of change in altitude between t=2 and t=2+h (s=2, h=h). Then, let h=0.

Average rate of change: $\frac{\Delta A}{\Delta t} = \frac{A(2+h) - A(2)}{h} = \frac{-2(2)(h) + (h)^2 + 8(h)}{h} = -2(2) + h + 8 = 4 + h$

Instantaneous: 4

Answer: 4 feet per minute.

c) If the balloon starts off at an altitude of 100 feet above ground at t=0, how far above ground will it be at time t=2 minutes?

The given formula allows us to compute the change in altitude from t=0 to t=2 (set s=0, h=2):

$$\Delta A = A(2) - A(0) = -2(0)(2) + (2)^2 + 8(2) = 4 + 16 = 20$$

Since the balloon starts off at A(0)=100 feet, and it goes up by 20 feet, it will be at A(2)=120 feet

Answer: At 120 feet.

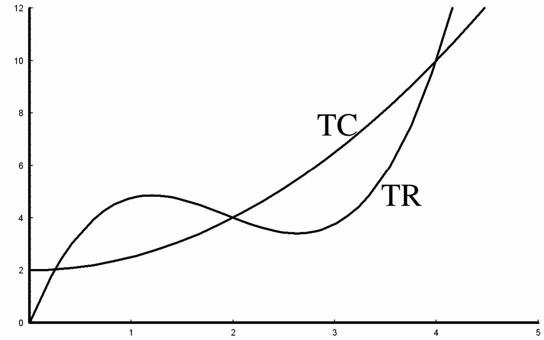
3) (20 Points)

To the right are the Total Revenue (TR) and Total Cost (TC) graphs for manufacturing and selling trinkets.

The corresponding formulas are:

$$TR(q) = q^3 - 5.75q^2 + 9.5q$$

$$TC(q) = 0.5q^2 + 2$$



where the quantity q is given in hundreds of trinkets, and both TR and TC are measured in hundreds of dollars.

a) Find formulas in terms of q for the Marginal Revenue and the Marginal Cost.

Use the differentiation laws to compute these.

ANSWER: $MR(q) = 3q^2 - 11.5q + 9.5$

$MC(q) = q$

MR and MC are measured in (circle one): *dollars* OR *hundreds of dollars.*

b) What quantity q between 0 to 4 hundred trinkets will result in the largest profit?

Set MR=MC and solve the resulting quadratic equation via the quadratic formula.

$$3q^2 - 11.5q + 9.5 = q$$

$$3q^2 - 12.5q + 9.5 = 0$$

$q=1$ or $q \approx 3.16$

Which one gives the max profit?! It must be a transition point from MR>MC to MR<MC, or, even easier, we can see from the TR/TC graphs above which root q has TR(q)>TC(q): the first (smaller) one. The other root corresponds to the minimum profit (max loss).

ANSWER: Profit is maximal for q = 1 hundred trinkets.

c) Find the longest interval over which the Total Revenue is increasing but the Marginal Revenue is decreasing.

Total Revenue increasing corresponds to MR>0. So we're looking for the longest interval over which MR is positive and decreasing. Since MR is a quadratic whose graph is a parabola that opens upward, it is positive (above the x-axis) outside its roots, and it decreases up to its vertex. Also, we're only interested in positive q (since it's a quantity!)

Roots: Quadratic formula on $3q^2 - 11.5q + 9.5 = 0$ *gives approximately q=1. 2 and q=2.63.*

The vertex is at q=1.9.

ANSWER: From $q = 0$ to $q = 1.2$ hundred trinkets.

d) For what quantity q larger than 1 hundred trinkets is the Total Revenue minimal?

The max/min of TR are among the roots of its derivative. Set TR'=0 We already computed these roots in part c). Looking at the provided graph, we see that the larger root (q=2.63) corresponds to the min of TR for q>1.

ANSWER: *q=2.63* hundred trinkets.