1. (12 points) Compute the derivatives. The correct answer with no supporting work receives *no points*. You do not have to simplify your final answer. Put a box around your final answer.

(a) (4 points) Find y' , if $y = \frac{3}{x^2} + \frac{\sqrt{x}}{5} + (3x^4)^2 + 4x^2x^4$.

$$y = 3x^{-2} + \frac{1}{5}x^{1/2} + 9x^8 + 4x^6$$

$$\boxed{y' = -6x^{-3} + \frac{1}{10}x^{-\frac{1}{2}} + 72x^7 + 24x^5}$$

(b) (4 points) Find $g'(x)$, if $g(x) = \frac{x^2 - 5x + 7}{2x^2}$.

$$g(x) = \frac{x^2}{2x^2} - \frac{5x}{2x^2} + \frac{7}{2x^2} = \frac{1}{2} - \frac{5}{2}x^{-1} + \frac{7}{2}x^{-2}$$

$$\boxed{g'(x) = \frac{5}{2}x^{-2} - 7x^{-3} = \frac{5}{2x^2} - \frac{7}{x^3}}$$

(c) (4 points) Find $\frac{dw}{dt}$, if $w = (t + 2t^{-1/2} + \frac{3}{t^{3/2}})\sqrt{t}$.

$$w = (t + 2t^{-1/2} + 3t^{-3/2})\, t^{1/2}$$

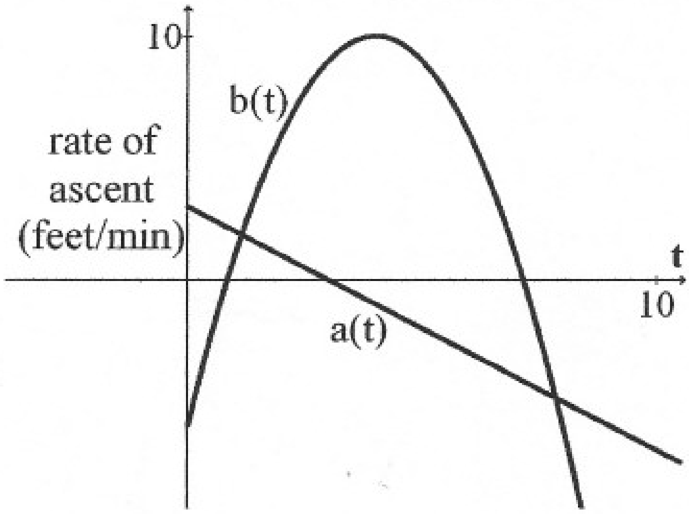
$$w = t^{3/2} + 2 + 3t^{-1}$$

$$\boxed{\frac{dw}{dt} = \frac{3}{2}t^{1/2} - 3t^{-2} = \frac{3}{2}\sqrt{t} - \frac{3}{t^2}}$$

2. (13 points)
Two balloons, A and B, are moving vertically straight up and down. At time $t = 0$, the balloons are both at a height of 100 feet. The formulas for the ***rate of ascent*** are given by:

$$a(t) = -t + 3 \quad \text{and} \quad b(t) = -t^2 + 8t - 6,$$

where t is in minutes and the rate of ascent is in feet/minute. The rate of ascent graphs are given at right.



(a) (3 points) Give the **rate of ascent** of balloon B at $t = 6$ minutes and determine if balloon B is rising or falling in **altitude** at this time.

$b(6) = -(6)^2 + 8(6) - 6 = -36 + 48 - 6$

ANSWER: rate of ascent = $\boxed{6}$ feet/minute

(clearly circle one) $\boxed{\text{RISING}}$ or FALLING

(b) (3 points) Find the time, between $t = 0$ and $t = 10$, when the **altitude** of balloon A is highest.

$a(t) = 0$

$-t + 3 = 0$

$t = 3$

altitude increases from 0 to 3
altitude decreases from 3 to 10 ∩

ANSWER: $t = \boxed{3}$ minutes

(c) (3 points) Which of the following best describes the motion of balloon B from $t = 0$ to $t = 10$? (clearly circle one and write your answer in the space provided.)

i. the altitude increases and then decreases
ii. the altitude decreases and then increases
iii. the altitude increases, then decreases, and then increases again
iv. the altitude decreases, then increases, and then decreases again

ANSWER: $\boxed{\text{iv}}$

(d) (4 points) Find the time, in the first 2 minutes, when the distance between the balloons is largest.

$a(t) = b(t)$

$-t + 3 = -t^2 + 8t - 6$

$t^2 - 9t + 9 = 0$

$t = \dfrac{9 \pm \sqrt{9^2 - 4(9)}}{2}$

$t = \dfrac{9 \pm \sqrt{45}}{2}$

$t = \cancel{7.8541}, 1.1459$

ANSWER: $t = \boxed{\approx 1.146}$ minutes

3. (13 points) Consider the function $H(x)$. You do not know the formula for $H(x)$, but you do know that the formula for the change in height of $H(x)$ from $x = p$ to $x = p + r$ is given by

$$H(p + r) - H(p) = 6r^2 - 7pr + 2r.$$

(a) (3 points) Find a formula involving k for $H(6 + k) - H(6)$.

$p = 6$

$r = k$

$H(6+k) - H(6) = 6k^2 - 7 \cdot 6 \cdot k + 2 \cdot k$

$= 6k^2 - 42k + 2k$

ANSWER: $H(6 + k) - H(6) = \boxed{6k^2 - 40k}$

(b) (3 points) If $H(6) = 70$, find the value of $H(8)$.

$p = 6$

$r = k = 2$

$H(8) - H(6) = 6(2)^2 - 40(2) = -56$

$H(8) - 70 = -56$

$H(8) = 14$

ANSWER: $H(8) = \boxed{14}$

(c) (3 points) Find the slope of the secant line to $H(x)$ from $x = 4$ to $x = 6$.

$\frac{H(6) - H(4)}{2} = ???$

$p = 4$

$r = 2$

$H(6) - H(4) = 6(2)^2 - 7(4)(2) + 2(2) = -28$

$\frac{H(6) - H(4)}{2} = -14$

ANSWER: slope = $\boxed{-14}$

(d) (4 points) Find a formula for $\frac{H(x + h) - H(x)}{h}$ and use it to find a formula for $H'(x)$.

$H(x+h) - H(x) = 6h^2 - 7xh + 2h$

$\frac{H(x+h) - H(x)}{h} = 6h - 7x + 2$

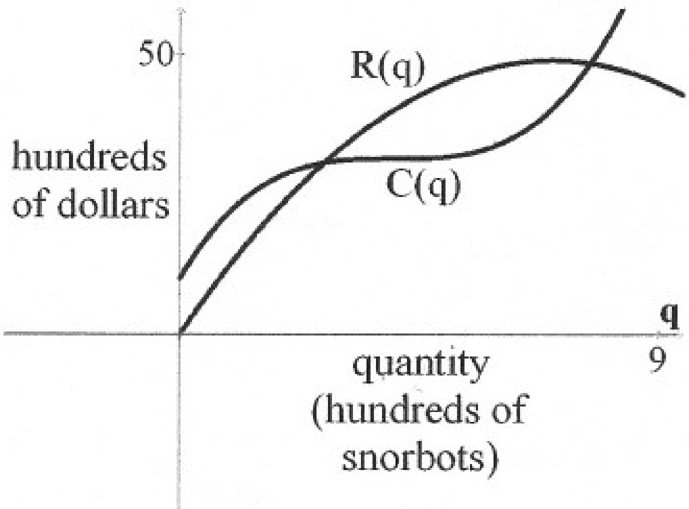
ANSWER: $\frac{H(x+h)-H(x)}{h} = \boxed{6h - 7x + 2}$ $H'(x) = \boxed{-7x + 2}$

4. (12 points)

You own a business that sells Snorbots. The functions for total revenue (TR) and total cost (TC) are given by

$$TR: \quad R(q) = -q^2 + 14q$$
$$TC: \quad C(q) = \frac{q^3}{3} - 4q^2 + 16q + 10$$

where $R(q)$ and $C(q)$ are in **hundreds** of dollars and q is in **hundreds** of Snorbots. The graphs of these functions are shown at right.



(a) (2 points) Use the derivative rules to find formulas for MR and MC.

ANSWER: $MR =$ $\boxed{-2q+14}$ and $MC =$ $\boxed{q^2-8q+16}$

(b) (3 points) Find all quantities at which the slope of the tangent line to the total revenue graph is equal to 4.

$-2q+14 = 4$

$-2q = -10$

$q = 5$

ANSWER: $q =$ $\boxed{5}$ **hundred Snorbots**

(c) (3 points) Between $q = 0$ and $q = 10$, find the largest interval when $R'(q)$ is positive.

vertex method: $q = \frac{-b}{2a} = -\frac{14}{2(-1)} = 7$ R(q) increasing

deriv. method: $-2q+14 = 0 \Rightarrow q = 7$

ANSWER: from $q=$ $\boxed{0}$ to $q =$ $\boxed{7}$ **hundred Snorbots**

(d) (4 points) Find the quantity where profit is maximum.
(Round the quantity to the nearest Snorbot.)

$MR = MC$

$-2q+14 = q^2-8q+16$

$0 = q^2-6q+2$

$q = \frac{6 \pm \sqrt{36-4(2)}}{2} = \frac{6 \pm \sqrt{28}}{2}$

5.64575, ~~0.354~~

Quantity: $\boxed{565}$ **Snorbots**