MATH 112 – EXAM I Hints and Answers
Version Alpha
Spring 2008

1. (a) (4 points) ANSWER: $MR(q) = -50q + 1250$, $MC(q) = 3q^2 - 120q + 601$
   (b) (3 points) HINT: Set $MR(q) = MC(q)$ and solve for q.
   ANSWER: $q = 30.440$ thousand Items
   (c) (4 points) HINT: $MR(q)$ is a linear function with "y"-intercept 1250 and a negative slope. Find the quantity at which $MR(q) = 100$ and the quantity at which $MR(q) = 250$. In between those two quantities, $MR(q)$ is between \$100 and \$250.
   ANSWER: from $q = 20$ to $q = 23$ thousand Items
   (d) (2 points) HINT: Either find the q-coordinate of the vertex of TR or find where MR (the slope of the tangent to TR) is equal to 0.
   ANSWER: $q = 25$ thousand Items
   (e) (3 points) HINT: Either find the vertex of MC or find where $MC'(q)$ (the slope of the tangent to MC) is equal to 0.
   ANSWER: from $q = 0$ to $q = 20$ thousand Items

2. (a) (4 points) HINT: Compute and simplify $\dfrac{D(5+h) - D(5)}{h}$.
   ANSWER: $\dfrac{D(5+h) - D(5)}{h} = -3h + 120$
   (b) (2 points) ANSWER: $D'(t) = -6t + 150$
   (c) (4 points) HINT: Set $D'(t) = 87$ and solve for t: $t = 10.5$. Compute $D(10.5)$ and divide by 10.5.
   ANSWER: 118.5 feet per second
   (d) (4 points) HINT: Find a formula, in terms of h, for $E(1+h) - E(1)$, the distance traveled on the interval from $t = 1$ to $t = 1 + h$:

$$E(1+h) - E(1) = \frac{5h}{2+h}.$$

   Set this formula equal to 4 and solve for h (you should get $h = 8$). But the question doesn't ask you for h, it asks for the interval, which was from $t = 1$ to $t = 1 + h = 9$.
   ANSWER: from $t = 1$ to $t = 9$
   (e) HINT: First find a formula for $\dfrac{E(m+h) - E(m)}{h}$, simplify, and see what happens if h approches 0.
   ANSWER: $E'(m) = \frac{10}{(m+1)^2}$

3. (4 points each)

   (a) ANSWER: $\dfrac{dy}{dx} = -\dfrac{5}{3}x^{-4/3} - \dfrac{1}{2}x^{-1/2}$
   (b) ANSWER: $\dfrac{ds}{dt} = 12t^3 - \dfrac{3}{2}t^2$

4. (a) HINT: Balloon A is rising the fastest when its rate of ascent is at its largest positive value. That will happen at the vertex of $a(t)$.
   ANSWER: $t = 3$

(b) HINT: Since the rate of ascent of Balloon A is positive and then negative, balloon A's altitude must increase and then decrease. The highest altitude that the balloon reaches must occur at the value of t at which the rate of ascent graph crosses the t-axis. To find this value, set $a(t)$ equal to 0 and solve for t.

ANSWER: $t = 7$

(c) EXPLANATION: The rate of ascent of Balloon B is negative and then positive. Whether the balloon is higher at $t = 3$ or $t = 4$ depends on whether the balloon increases or decreases between these two times. So, it is necessary to determine where the rate graph crosses the t-axis. To find this time, set $b(t)$ equal to 0 and solve for t: $t = 4$. This means that the balloon is falling from $t = 0$ to $t = 4$. In particular, between $t = 3$ and $t = 4$, the balloon falls and is therefore higher at $t = 3$ than $t = 4$.