VI

1. (12 points) The correct answer with no supporting work receives ***no points***. You do not have to simplify your final answer. Put a box around your final answer.

(a) (4 points) Find $\frac{dy}{dx}$, if $y = (4x^2 + 1)(5 - x^3) + 2x$.

$y = -4x^2x^3 + 20x^2 + 5 - x^3 + 2x$

$y = -4x^5 + 20x^2 + 5 - x^3 + 2x$

$\boxed{\frac{dy}{dx} = -20x^4 + 40x - 3x^2 + 2}$

(b) (4 points) Find $f'(t)$, if $f(t) = \frac{3t^5 + 4 - \sqrt{t}}{2t^3}$.

$f(t) = \frac{3t^5}{2t^3} + \frac{4}{2t^3} - \frac{\sqrt{t}}{2t^3}$

$f(t) = \frac{3}{2}t^2 + 2t^{-3} - \frac{1}{2}\frac{t^{1/2}}{t^3}$

$f(t) = \frac{3}{2}t^2 + 2t^{-3} - \frac{1}{2}t^{-5/2}$

$\boxed{f'(t) = 3t - 6t^{-4} + \frac{5}{4}t^{-7/2}}$

OTHER CORRECT ANSWERS

$= 3t - \frac{6}{t^4} + \frac{5}{4t^{7/2}}$

$= 3t - 6t^{-4} + 1.25t^{-3.5}$

$= 3t - \frac{6}{t^4} + \frac{1.25}{t^{3.5}}$

(c) (4 points) Find the slope of the tangent line to $g(x) = \frac{x^3}{3} + \frac{5}{x^2} + 6\sqrt[3]{x^2}$ at $x = 1$.

$g(x) = \frac{1}{3}x^3 + 5x^{-2} + 6x^{2/3}$

$g'(x) = \frac{1}{3}3x^2 - 10x^{-3} + 6\,\frac{2}{3}x^{-1/3}$

$g'(x) = x^2 - \frac{10}{x^3} + \frac{4}{\sqrt[3]{x}}$

$g'(1) = 1 - 10 + 4 = \boxed{-5}$

$\boxed{\text{VI}}$

2. (12 points) Let $f(x) = 5x - 3x^2 + 1$.

(a) (4 pts) Write out a formula, in terms of h, for

$$\frac{f(x+h) - f(x)}{h}.$$

Simplify as much as possible.

$$\frac{f(x+h)-f(x)}{h} = \frac{[5(x+h) - 3(x+h)^2+1] - [5x-3x^2+1]}{h}$$

$$= \frac{\cancel{5x} + 5h - 3(x^2+2xh+h^2) + \cancel{1} - \cancel{5x} + 3x^2 - \cancel{1}}{h}$$

$$= \frac{5h - \cancel{3x^2} - 6xh - 3h^2 + \cancel{3x^2}}{h}$$

$$= 5 - 6x - 3h$$

ANSWER: $\boxed{\frac{f(x+h)-f(x)}{h} = 5-6x-3h}$

(b) (2 pts) Find the formula for the derivative $f'(x)$.

Let $h \to 0 \Rightarrow f'(x) = 5-6x$

Check $f'(x) = 5-6x$ ✓

ANSWER: $\boxed{f'(x) = 5-6x}$

3. Consider the function $g(x)$. You do not know the formula for $g(x)$, but you do know that the formula for the slope of the secant line to $g(x)$ from $x = m$ to $x = m + h$ is given by

$$\frac{g(m+h) - g(m)}{h} = \frac{3}{(m+1)(m+h+1)}.$$

(a) (3 pts) Find a formula involving k for $g(2+k) - g(2)$.

Let $m=2$, $h=k$

$$\frac{g(2+k)-g(2)}{k} = \frac{\cancel{3}}{\cancel{(2+1)}(2+k+1)} = \frac{1}{3+k} \Rightarrow g(2+k)-g(2) = \frac{k}{3+k}$$

ANSWER: $g(2 + k) - g(2) = \boxed{\frac{k}{3+k}}$

(b) (3 pts) Find the slope of the tangent line to $g(x)$ at $x = 5$. $m=5$

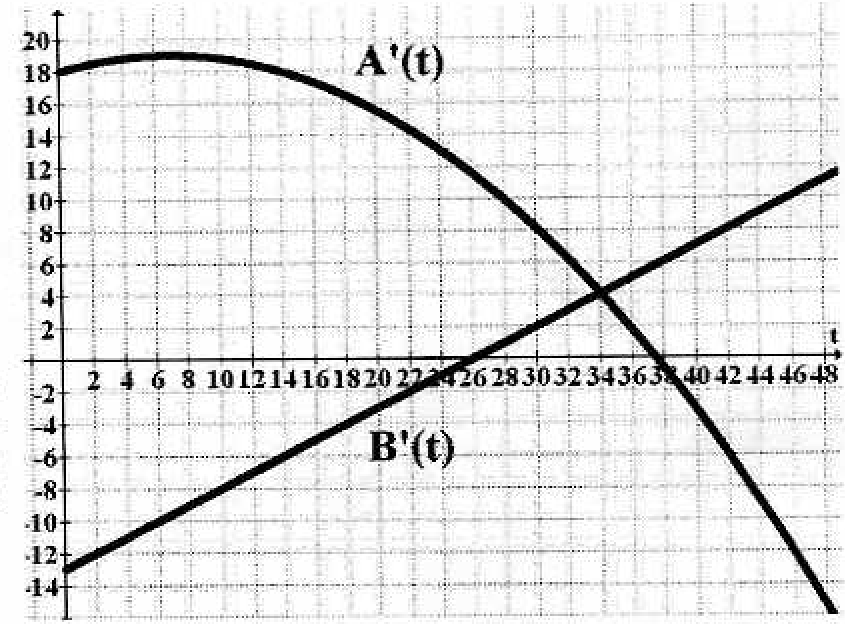
$$\frac{g(5+h)-g(5)}{h} = \frac{3}{(5+1)(5+h+1)} = \frac{3}{6(6+h)}$$

Let $h \to 0 \Rightarrow \frac{3}{6\cdot 6} = \frac{3}{36} = \frac{1}{12}$

ANSWER: slope = $\boxed{\frac{1}{12}}$

4. (14 points)
Two balloons, A and B, are moving vertically straight up and down.
At time $t = 0$, the balloons are both at a height of 200 feet. The graphs of the ***RATE OF ASCENT*** (i.e. speed) are given, where t is in minutes and the rate of ascent is in feet/minute.

Use the graphs to estimate the answers to the following questions. Be as accurate as possible and briefly explain your work (a sentence or short phrase is sufficient).



(a) (3 pts) Find the time when Balloon A is at it's highest altitude.

Balloon A rises (increases altitude) from $t=0$ to $t=38$
and it falls (decreasing altitude) after $t=38$.

ANSWER: $t =$ $\boxed{38}$ min

(b) (3 pts) Give the approximate value of $\frac{B(10.0001) - B(10)}{0.0001}$. $\approx$ slope of the tangent line to $B(t)$ at $t=10$

$B'(10) = -8$

ANSWER: $\frac{B(10.0001)-B(10)}{0.0001} \approx$ $\boxed{-8}$ ft/min

(c) (3 pts) Find the time when Balloon A is above Balloon B by the greatest distance.

Balloon A rises faster than B from $t=0$ to $t=34$.
After which B starts to go faster and the dist. between gets smaller.

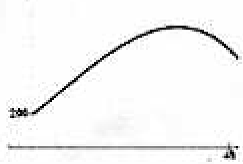
ANSWER: $t =$ $\boxed{34}$ min

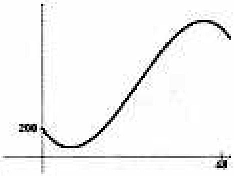
(d) (3 pts) Give the largest interval when balloon $\underbrace{B \text{ is rising (increasing)}}_{B' \text{ positive} \leftarrow \text{ after } 26}$ and the vertical distance between the $\underbrace{\text{balloons is getting larger,}}_{A'(t) > B'(t)}$

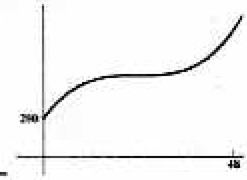
before 34

ANSWER: $t =$ $\boxed{26}$ to $t =$ $\boxed{34}$

(e) (2 pts) Of the four graphs below, in the blanks provided label the altitude (i.e. distance) graph of Balloon A and the altitude graph of Balloon B. (Two will be left blank).

A 

__ 

__ 

B 

VI

5. (12 points) You own a business that sells Toy Dinosaurs. The functions for total revenue (TR) and total cost (TC) are given by

$$TR: \quad R(q) = 110q - 3q^2 \quad 70$$
$$TC: \quad C(q) = \frac{q^3}{30} - \frac{3q^2}{2} + 40q + 10$$

where $R(q)$ and $C(q)$ are in **hundreds** of dollars and q is in **hundreds** of toy dinosaurs. Keep all answer accurate to the nearest Toy Dinosaur or the nearest cent.

(a) (2 pts) Give the formulas for $MR(q) = R'(q)$ and $MC(q) = C'(q)$.

$mc(q) = \frac{3}{30}q^2 - 3q + 40$

ANSWER: $MR(q) =$ $120 - 6q$

$MC(q) =$ $\boxed{mc(q) = \frac{1}{10}q^2 - 3q + 40}$

(b) (3 pts) Find the slope of variable cost $VC(q)$ at $q = 4$ hundred Toy Dinosaurs.

$vc'(q) = mc(q)$ $\qquad vc(q) = \frac{q^3}{30} - \frac{3q^2}{2} + 40q$

$mc(4) = \frac{1}{10}(4)^2 - 3(4) + 40 = 29.6$

ANSWER: $\boxed{29.60}$ dollars per Toy Dinosaur

(c) (3 pts) Find an interval of length 3 hundred Toy Dinosaurs over which MR changes from positive to zero to negative.

$mr(q) = 0 \Rightarrow 110 - 6q = 0 \Rightarrow q = 110$ TR looks like

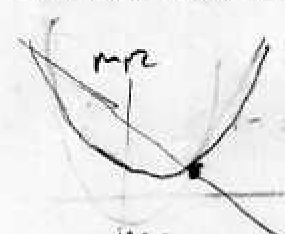
$q = \frac{110}{6} = \frac{55}{3} = 18.\overline{3}$

mr looks like

ANY INTERVAL OF LENGTH 3 INCLUDING $18.\overline{3}$

ANSWER: $q =$ $\boxed{17}$ to $q =$ $\boxed{20}$ hundred Toy Dinosaur

(d) (4 pts) Find the quantity where profit is maximum.
(Give your answer to the **nearest Toy Dinosaur**.)



$mr(q) = mc(q)$

$110 - 6q = \frac{1}{10}q^2 - 3q + 40$

$0 = \frac{1}{10}q^2 + 3q - 70$

$q = \dfrac{-3 \pm \sqrt{9 - 4(\frac{1}{10})(-70)}}{2(1/10)}$

$q = \dfrac{-3 \pm \sqrt{37}}{0.2} = \begin{matrix} -45.4138 \\ \text{or} \\ 15.413812 \end{matrix}$

Quantity: $\boxed{1541}$ **Toy Dinosaurs**