

Math 112 Midterm 1 Solutions, Spring 2011 (version 1)

1. a) $f(x) = (2x + 1)(7 - x^2) = -2x^3 - x^2 + 14x + 7 \Rightarrow f'(x) = \boxed{-6x^2 - 2x + 14}$

b) $y = 5\sqrt[3]{x} + x\sqrt{x} = 5x^{\frac{1}{3}} + x^{\frac{3}{2}} \Rightarrow \frac{dy}{dx} = \boxed{\frac{5}{3}x^{-\frac{2}{3}} + \frac{3}{2}x^{\frac{1}{2}} = \frac{5}{3\sqrt[3]{x^2}} + \frac{3\sqrt{x}}{2}}$

c) $g(x) = \frac{x^5 + 7}{x^2} - 0.77 = x^3 + 7x^{-2} - 0.77 \Rightarrow g'(x) = \boxed{3x^2 - 14x^{-3} = 3x^2 - \frac{14}{x^3}}$

2. a) In order: none, B, D, C, none, A

b) The slope of the secant line from $x = s$ to $x = s + r$ is:

$$\frac{f(s+r) - f(s)}{r} = \frac{2sr + r}{(s+3)(s+r)} \times \frac{1}{r} = \frac{(2s+1)r}{(s+3)(s+r)} \times \frac{1}{r} = \frac{2s+1}{(s+3)(s+r)}$$

The slope of the tangent line at $x = s$ is obtained from this by letting r go to zero:

$$f'(s) = \frac{2s+1}{(s+3)(s)}$$

Evaluating at $s = 7$:

$$f'(7) = \frac{2(7)+1}{(7+3)7} = \boxed{\frac{15}{70} \approx 0.21}$$

3. a) $\frac{B(3.1)-B(3)}{0.1} \approx B'(3) = \boxed{0.5}$

b) $\boxed{\text{from } t = 0 \text{ to } t = 5 \text{ minutes}}$

c) at $\boxed{t = 7}$ minutes

d) from $t = 0$ to $t = 5$, Balloon A is: $\boxed{\text{descending}}$, & Balloon B is: $\boxed{\text{first ascending then descending}}$

4. a) $MR(q) = \boxed{3q^2 - 11.5q + 9.5}$, $MC(q) = \boxed{\frac{6}{5}q = 1.2q}$, Units for both: $\boxed{\text{Dollars}}$

b) Setting $MR = MC$ results in the equation: $3q^2 - 12.7q + 9.5 = 0$.

Quadratic Formula: $q \cong 0.97$ and $q \cong 3.26$

TR is above TC at the first one only (other one is max loss). Answer: at: $\boxed{q \cong 0.97}$ hundred trinkets

c) TR has a horizontal tangent line when $TR'(q) = 3q^2 - 11.5q + 9.5 = 0$

Quadratic Formula: $\boxed{q \cong 1.2 \text{ and } 2.63}$ hundred trinkets