MATH 112
Sample Exam I Solutions

1. (a) Take $r = 3$ and $h = 0.5$. Then,

$$f(3.5) - f(3) = \frac{0.5}{(3 + 0.5 - 4)(3 - 4)} = \frac{0.5}{(-0.5)(-1)} = 1.$$

(b) The slope of the secant line from $x = 2$ to $x = 2.05$ is $\frac{f(2.05) - f(2)}{0.05}$. So, we take $r = 2$ and $h = 0.05$. Then,

$$f(2.05) - f(2) = \frac{0.05}{(2 + 0.05 - 4)(2 - 4)} = \frac{0.05}{(-1.95)(-2)} = 0.0128205.$$

To get the slope we must divide this quantity by 0.05:

$$\frac{f(2.05) - f(2)}{0.05} = \frac{0.0128205}{0.05} = 0.2564.$$

(c) Take $r = 1$. Then, $f(1 + h) - f(1) = \frac{h}{(1 + h - 4)(1 - 4)} = \frac{h}{(h - 3)(-3)}$.

$$\frac{f(1 + h) - f(1)}{h} = \frac{\frac{h}{(h-3)(-3)}}{\frac{h}{1}} = \frac{h}{(h - 3)(-3)} \cdot \frac{1}{h} = \frac{1}{(h - 3)(-3)}.$$

We want to know when this is equal to 12:

$$\frac{1}{(h - 3)(-3)} = 12 \Rightarrow 1 = (-36)(h - 3) = -36h + 108$$

$$\Rightarrow 36h = 107 \Rightarrow h = \frac{107}{36} = 2.9722.$$

(d)

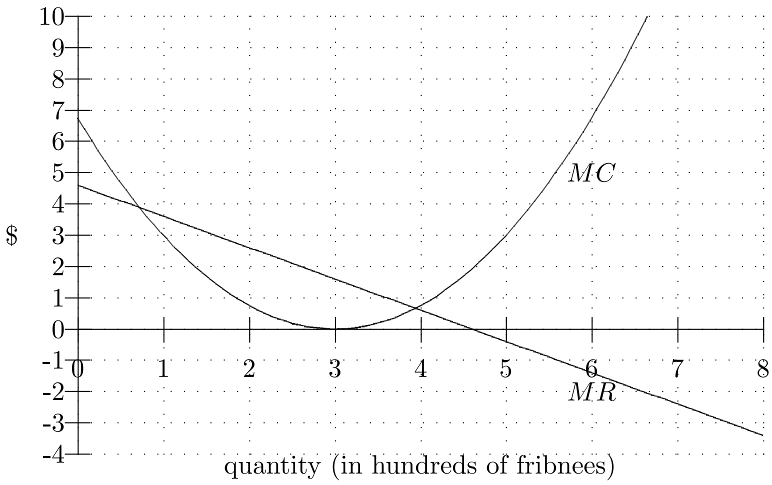
$$\frac{f(r + h) - f(r)}{h} = \frac{\frac{h}{(r+h-4)(r-4)}}{\frac{h}{1}} = \frac{h}{(r + h - 4)(r - 4)} \cdot \frac{1}{h} = \frac{1}{(r + h - 4)(r - 4)}.$$

As h gets close to 0, this gets close to $\frac{1}{(r - 4)^2}$. So, $f'(r) = \frac{1}{(r - 4)^2}$.

(e) $f'(3.8) = \frac{1}{(3.8 - 4)^2} = \frac{1}{(-.2)^2} = \frac{1}{0.04} = 25.$

2. (a) MR and MC are both positive and decreasing over the interval from 0 to about 3.

(b) $MR : R'(q) = -q + 4.6$ and $MC : C'(q) = 0.75q^2 - 4.5q + 6.75$



(c) Profit is maximized when $MR = MC$.

$$-q + 4.6 = 0.75q^2 - 4.5q + 6.75$$
$$0 = 0.75q^2 - 3.5q + 2.15$$
$$q = 3.94 \text{ or } 0.73$$

Do both these values of q maximize profit? We can see from the graphs of TR and TC that profit is bigger at $q = 3.94$ than at $q = 0.73$. So, profit is maximized when $q = 3.94$ or when we sell 394 fribnees.

(d) TR is greatest when $R(q)$ has a horizontal tangent line. This occurs when MR is 0.

$$-q + 4.6 = 0 \Rightarrow q = 4.6.$$

So, TR is greatest when we sell 460 fribnees.

(e) Since the formula for MC is a quadratic, we can use the vertex formula to find the q that gives the smallest MC. The vertex of $C'(q)$ is at $q = \dfrac{-(-4.5)}{2(0.75)} = 3$. MC is lowest when we sell 300 fribnees.

(f) If Fixed Cost was \$200, the graph of TC would be shifted down 4 units. The new TC function would be

$$C(q) = 0.25q^3 - 2.25q^2 + 6.75q + 2.$$

So, the MC function would be the same as before:

$$C'(q) = 0.75q^2 - 4.5q + 6.75.$$

The answers to parts (c) and (e), therefore, would not change, since they depended only on MC and not on TC.

3. (a) The derived graph of $f(x)$ is above the x-axis precisely when the graph of $f(x)$ is increasing. This is on the interval from about $x = 7$ to $x = 15$.

(b) Since the given graph of $g'(x)$ starts out below the x-axis, crosses the x-axis, and then goes above the x-axis, we need an interval on which $g(x)$ is decreasing, has a horizontal tangent, and then becomes increasing. One such interval is from $x = 9$ to $x = 15$. (Other intervals are also correct.)

(c) $f(2+h) = 0.1h^2 - h + 6.5$ and $f(2) = 6.5$. So, $f(2+h) - f(2) = 0.1h^2 - h$. Thus,

$$\frac{f(2+h) - f(2)}{h} = (0)h^2 + (0.1)h + (-1).$$

(d) This is not possible since the slope of the tangent to $f(x)$ is always increasing. That means $f'(x)$ is always increasing.