MATH 112 - Winter 2005
Exam 1, Version 1 - Hints and Answers

1. (a) (3 points) HINT: $f'(x)$ crosses the x-axis when $f(x)$ has a horizontal tangent line.
   ANSWER: $x = 2$, 5, and 8

   (b) (3 points) HINT: $g(x)$ has a horizontal tangent line when $g'(x)$ crosses the x-axis
   ANSWER: $x = 3$, 6, and 10

   (c) (2 points) HINT: $f'(x)$ is above the x-axis when $f(x)$ is increasing
   ANSWER: from 0 to 2 and from 5 to 8

   (d) (2 points) HINT: $g(x)$ decreases when $g'(x)$ is below the x-axis
   ANSWER: from 0 to 3 and from 6 to 10

   (e) (1 point) ANSWER: iv

2. (4 points each)

   (a) HINT: $y = x^{1/3} - 4x^{-2/3}$
   ANSWER: $\dfrac{dy}{dx} = \dfrac{1}{3}x^{-2/3} - 4\left(-\dfrac{2}{3}x^{-5/3}\right)$

   (b) HINT: $R(q) = q^5 + 2q^2$
   ANSWER: $R'(q) = 5q^4 + 4q$

   (c) HINT: $s = 5t^5 - 4t + 1$
   ANSWER: $\dfrac{ds}{dt} = 25t^4 - 4$

3. (a) (3 points) HINT: You're looking for $\dfrac{A(4+0.5) - A(4)}{0.5}$. So, take $t = 4$ and $m = 0.5$ in the given formula.
   ANSWER: 11.5 thousand gallons per hour

   (b) (4 points) HINT: Take $t = 6$ and $m = h$ and use the given formula to compute $\dfrac{A(6+h) - A(6)}{h}$. Multiply both sides by h to get $A(6+h) - A(6) = 8h - h^2$. Set this equal to 16 and solve for q.
   ANSWER: $h = 4$

   (c) (4 points) HINT: You want to compute $A'(8.73)$. You have a formula for the slope of the secant line through $A(t)$ at t and $t + m$. To get $A'(t)$, let m go to zero: $A'(t) = 20 - 2t$.
   ANSWER: $A'(8.73) = 2.54$ thousand gallons per hour

(d) (4 points) HINT: You know that $A(2) = 6$ and you want $A(5)$. Take $t = 2$ and $m = 3$ and use the given formula to compute $\dfrac{A(5) - A(2)}{3}$. Multiply by 3 to get $A(5) - A(2) = 39$. Now use the fact that $A(2) = 6$ and solve for $A(5)$.
ANSWER: $A(5) = 45$ thousand gallons

4. (4 points each)

(a) HINT: Speed is the derivative of distance. So, Bert's speed is $B'(t) = -2t + 16$ and Ernie's is $E'(t) = 2t$. You need to solve the equation $E'(t) = B'(t) + 13.44$ for t.
ANSWER: $t = 7.36$ minutes

(b) HINT: $B(1 + h) = -(1 + h)^2 + 16(1 + h) = -h^2 + 14h + 15$ and $B(1) = 15$.
ANSWER: $B(1 + h) - B(1) = (-1)h^2 + (14)h + (0)$

(c) HINT: Solve $B'(t) = 8.86$ for t: $t = 3.57$. Compute $E(3.57)$.
ANSWER: 12.7449 feet (rounding to 12.74 feet is OK)