

MATH 112
Exam I
January 31, 2006

Name _____

Student ID # _____

Section _____

HONOR STATEMENT

“I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam.”

SIGNATURE: _____

1	16	
2	17	
3	17	
Total	50	

- Your exam should consist of this cover sheet, followed by 3 problems on 5 pages. Check that you have a complete exam.
- Turn your cell phone OFF and put it away for the duration of the exam.
- Unless otherwise indicated, you must show your work. The correct answer with no supporting work may result in no credit.
- If you use a guess-and-check method when an algebraic method is available, you may not receive full credit.
- Put your name on your sheet of notes and turn it in with the exam.

GOOD LUCK!

1. (16 points)

(a) If $f(x) = \frac{1}{4}x^8 - \frac{1}{3}x^6 + \frac{7}{\sqrt{x}} + 9$, find $f'(x)$.

ANSWER: $f'(x) =$ _____

(b) If $w = \frac{4z - 11}{z^6}$, find $\frac{dw}{dz}$.

ANSWER: $\frac{dw}{dz} =$ _____

(c) If $h(t) = (t^2 + 1)(t + \frac{3}{t})$, find the slope of the tangent line to $h(t)$ at $t = 1$.

slope= _____

(d) If total revenue (in dollars) for selling q Things is given by the formula

$$TR(q) = -0.001q^2 + 2.8q,$$

find all values of q at which marginal revenue is 0.254 dollars. (Use the fact that $MR(q) = TR'(q)$.)

$q =$ _____

2. (17 points) You sell Items. The formula for total cost is given by some function $TC(q)$, where q is measured in thousands of Items and TC is in thousands of dollars. The formula for the *change* in TC from q_1 to q_2 thousand Items is

$$TC(q_2) - TC(q_1) = q_2^2 - q_1^2 + 16(q_2 - q_1).$$

- (a) Compute $\frac{TC(7) - TC(2)}{5}$.

ANSWER: _____

- (b) If you produce 10 thousand Items, your total cost is 326 thousand dollars (i.e., $TC(10) = 326$). What is the value of your fixed cost? (Recall: $FC = TC(0)$.)

ANSWER: $FC =$ _____ thousand dollars

- (c) Since we are measuring quantity in thousands of Items, the *definition* of marginal cost as the change in total cost that occurs when you increase production by 1 Item (0.001 thousand Items) can be written

$$MC(q) = TC(q + 0.001) - TC(q).$$

Use this definition to compute the marginal cost at $q = 4$ thousand Items. Include units in your answer.

ANSWER: $MC(4) =$ _____

(This problem is continued on the next page.)

Again, the formula for the change in TC is given by:

$$TC(q_2) - TC(q_1) = q_2^2 - q_1^2 + 16(q_2 - q_1).$$

(c) Compute the formula for the derivative $TC'(q)$.

ANSWER: $TC'(q) =$ _____

(d) Use your answer to part (d) and the approximation $MC(q) = TC'(q)$ to compute the cost to produce the 7,001st Item.

ANSWER: _____ dollars

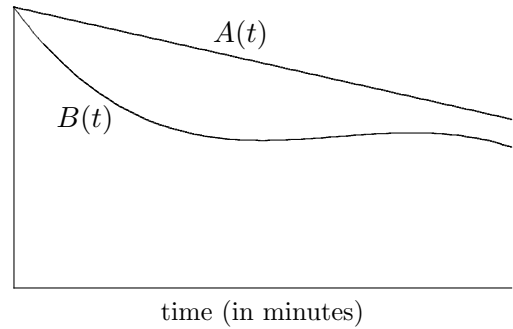
3. (17 points)

The graph at right shows the altitude versus time graphs for two balloons. The altitude (in feet) of balloon A after t minutes is given by the formula

$$A(t) = 500 - 20t$$

and the altitude (in feet) of balloon B after t minutes is given by the formula

$$B(t) = -t^3 + 19.5t^2 - 120t + 500.$$



- (a) Find the lowest altitude that the A -balloon reaches in the time interval from $t = 3$ to $t = 18$ minutes.

ANSWER: _____ feet

- (b) Find the time in the first five minutes at which the two balloons are farthest apart.

ANSWER: $t =$ _____ minutes

This problem is continued on the next page.

Those formulas again are:

$$A(t) = 500 - 20t \text{ and } B(t) = -t^3 + 19.5t^2 - 120t + 500.$$

- (c) Find the largest interval on which the B -balloon is rising.

ANSWER: from $t =$ _____ to $t =$ _____ minutes

- (d) At what time does the B -balloon reach its fastest upward speed?

ANSWER: $t =$ _____ minutes

- (e) What is the B -balloon's fastest downward speed on the interval from $t = 1$ to $t = 4$?

ANSWER: _____ feet per minute