

Math 112 B and C - Winter 2006

Exam 1

January 31, 2006

Name: Instructor Solution Key

Section: Version 1

Student ID Number: _____

TA's Name: _____

1	9	
2	11	
3	10	
4	9	
5	11	
Total	50	

- You are allowed to use a calculator and one **hand-written** 8.5 by 11 inch page of notes. Put your name on your sheet of notes and turn it in with the exam.
- Check that your exam contains all the problems listed above.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- There are multiple versions of the exam. Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 50 minutes to complete the exam.

GOOD LUCK!

1. (9 points) Compute the derivatives. The correct answer with no supporting work receives *no* points. You do not have to simplify your final answer.

(a) (3 points) Find y' , if $y = x(x^2 + 3x + 1)$.

$$y = x^3 + 3x^2 + x \quad \leftarrow \begin{cases} \text{need to see} \\ \text{this step} \end{cases}$$

$$y' = 3x^2 + 6x + 1$$

$$\text{ANSWER: } y' = \boxed{3x^2 + 6x + 1}$$

(b) (3 points) Find $\frac{dz}{dw}$, if $z = \frac{2w^3 + 5w^2 - w}{w}$.

$$z = 2w^2 + 5w - 1 \quad \leftarrow \begin{cases} \text{need to see} \\ \text{this step} \end{cases}$$

$$\frac{dz}{dw} = 4w + 5$$

$$\text{ANSWER: } \frac{dz}{dw} = \boxed{4w + 5}$$

(c) (3 points) Find $g'(t)$, if $g(t) = \sqrt{t} \left(3t - \frac{1}{t} \right)$.

$$g(t) = 3t^{1.5} - t^{-0.5} \quad \leftarrow \begin{cases} \text{need to see} \\ \text{this step} \end{cases}$$

$$g'(t) = 4.5t^{0.5} + 0.5t^{-1.5}$$

$$\text{ANSWER: } g'(t) = \boxed{4.5t^{0.5} + 0.5t^{-1.5}}$$

2. (11 points) Consider the function $g(x)$. You do not know the formula for $g(x)$, but you do know the formula for the slope of the secant line through $g(x)$ at $x = m$ and $x = m + h$ is given by:

$$\frac{g(m+h) - g(m)}{h} = 2m + 5mh - 7.$$

- (a) (3 points) Compute and simplify

$$\frac{g(3.1) - g(3)}{0.1}$$

$$m = 3 \quad h = 0.1$$

$$\begin{aligned} \frac{g(3+0.1) - g(3)}{0.1} &= 2(3) + 5(3)(0.1) - 7 \\ &= 6 + 1.5 - 7 \end{aligned}$$

$$\text{ANSWER: } \frac{g(3.1) - g(3)}{0.1} = \boxed{0.5}$$

- (b) (4 points) Find a formula involving h for

$$g(2+h) - g(2).$$

$$\begin{aligned} g(2+h) - g(2) &= h [2m + 5mh - 7] \\ &= h [4 + 10h - 7] \\ &= h [10h - 3] \end{aligned}$$

$$\text{ANSWER: } g(2+h) - g(2) = \boxed{10h^2 - 3h}$$

- (c) (4 points) Find the slope of the tangent line to $g(x)$ at $x = 1.4$.

$$\text{Let } h \rightarrow 0, \quad g'(x) = 2x - 7$$

$$\begin{aligned} g'(1.4) &= 2(1.4) - 7 \\ &= 2.8 - 7 = -4.2 \end{aligned}$$

$$\text{ANSWER: } g'(1.4) = \boxed{-4.2}$$

okay if you used 3.4

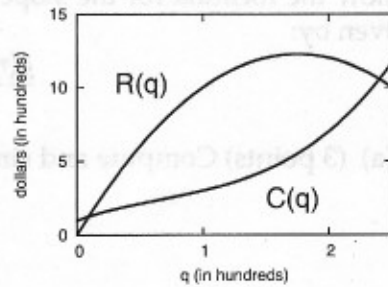
3. (10 points)

You own a business that sells umbrellas. The functions for total revenue (TR) and total cost (TC) are given by

$$TR: R(q) = -4q^2 + 14q$$

$$TC: C(q) = q^3 - 2q^2 + 3q + 1$$

where $R(q)$ and $C(q)$ are in hundreds of dollars and q is in hundreds of umbrellas. The graphs of these functions are shown at right.



(a) (4 points) Assume that Marginal Revenue is the slope of the tangent line to Total Revenue and Marginal Cost is the slope of the tangent line to the Total Cost.

Use the derivative rules to find formulas for MR and MC .

$$MR = R'(q) = -8q + 14$$

$$MC = C'(q) = 3q^2 - 4q + 3$$

ANSWER: $MR = -8q + 14$ and $MC = 3q^2 - 4q + 3$

(b) (6 points) Find the quantity where profit is maximum and give the maximum profit. (Round the quantity to the nearest umbrella and the profit to the nearest cent.)

$$MR = MC$$

$$-8q + 14 = 3q^2 - 4q + 3$$

$$0 = 3q^2 + 4q - 11$$

$$q = \frac{-4 \pm \sqrt{4^2 - 4(3)(-11)}}{2(3)}$$

$$q = 1.360921$$

or

$$q = -2.69425$$

$$P(q) = R(q) - C(q)$$

$$P(1.36) = R(1.36) - C(1.36)$$

$$= 11.6416 - 3.896256$$

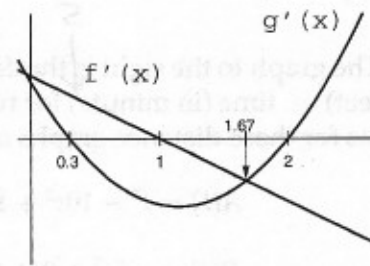
$$= 7.745344$$

ANSWER: Quantity: 136 umbrellas Maximum Profit: \$ 774.53

4. (9 points)

To the right is the *derived* graphs of two functions. The "original graphs" from which these graphs came are not shown. The original graphs are given by the functions

Original graphs: $y = f(x)$, $y = g(x)$.



(a) (3 points) Find the value of x between $x = 0$ and $x = 2$ where the value of $f(x)$ is highest.

$f(x)$: horizontal tangent at $x=1$
 increasing before
 decreasing after

ANSWER: $x =$

1

(b) (3 points) Give the largest interval where $f(x)$ is increasing and $g(x)$ is decreasing.

$f'(x)$ positive
 $g'(x)$ negative

ANSWER: from $x =$ 0.3 to $x =$ 1

(c) (3 points) Which of the following is true about the graph of $g(x)$ on the interval from $x = 1.5$ to $x = 2.2$?

- i. It is always decreasing.
- ii. It decreases, has a horizontal tangent, and then increases.
- iii. It is always increasing.
- iv. It increases, has a horizontal tangent, and then decreases.

ANSWER:

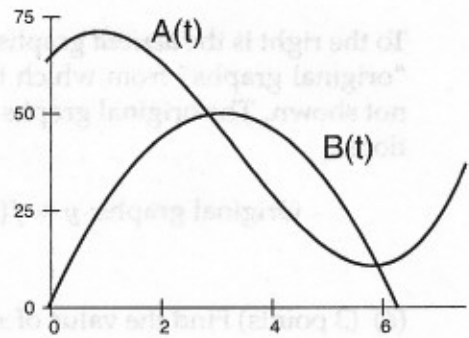
ii

5. (11 points)

The graph to the right is the distance off the ground (in feet) vs. time (in minute) for two balloons. The formulas for these distance graphs are

$$A(t) = t^3 - 10t^2 + 15t + 65, \text{ and}$$

$$B(t) = -5t^2 + 31t + 1.95.$$



- (a) (3 points) Give the largest interval between $t = 0$ and $t = 6$ where the derived graph $B'(t)$ is positive.

0 to "peak" of $B(t)$
 Vertex of $B(t) = t = \frac{-31}{2(-5)} = 3.1$ } or use derivative

ANSWER: from $t = 0$ minutes to $t = 3.1$ minutes

- (b) (4 points) Write down an equation you would solve to find the value of t at which the speed of Balloon A is the same as the speed of Balloon B. Do not solve the equation. Write it in the form $(\)t^3 + (\)t^2 + (\)t + (\) = 0$. (Some of the coefficients may be 0.)

$$A'(t) = B'(t)$$

$$3t^2 - 20t + 15 = -10t + 31$$

$$3t^2 - 10t - 16 = 0$$

or $-3t^2 + 10t + 16 = 0$
 or any other nonzero multiple

ANSWER: $(3)t^3 + (-10)t^2 + (-16)t + 0 = 0$

- (c) (4 points) Find the instantaneous speed of Balloon B at time $t = 1.1$ minutes.

$$B'(t) = -10t + 31$$

$$B'(1.1) = -10(1.1) + 31 = -11 + 31$$

ANSWER: speed = 20 feet/minute