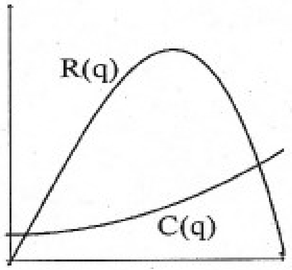
1. (12 points) The formulas and graphs for Total Revenue and Total Cost are given:

$$TR: R(q) = -0.2q^3 + \frac{q^2}{2} + 5.5q$$

$$TC: C(q) = 0.25q^2 + 2$$



(a) (4 points) Use the derivative rules to give formulas for Marginal Revenue and Marginal cost.

$MR(q) =$ $-0.6q^2 + q + 5.5$

$MC(q) =$ $0.5q$

(b) (4 points) Find all positive quantities at which the graph of the Profit function has a horizontal tangent line.

MR = MC

$-0.6q^2 + q + 5.5 = 0.5q$

$0 = 0.6q^2 - 0.5q + 5.5$

$q = \dfrac{0.5 \pm \sqrt{(-0.5)^2 - 4(0.6)(-5.5)}}{2(0.6)}$

$q = 3.47285347$ or -2.6395201

ANSWER: $\boxed{q = 3.47}$

(c) (4 points) The Average Cost is defined by

$$AC(q) = \frac{C(q)}{q} = \cancel{\frac{0.2q^2+1}{q}} = \frac{0.25q^2+2}{q}$$

Find the derivative of the Average Cost and give the slope of the average cost graph at $q = 3$.

$AC = 0.25q + \frac{2}{q} = 0.25q + 2q^{-1}$

$AC'(q) = 0.25 - 2q^{-2}$

$\boxed{AC'(q) = 0.25 - 2q^{-2} = 0.25 - \frac{2}{q^2}}$

$\boxed{\text{"Slope of AC at q=3"} = AC'(3) = 0.02\overline{7} \approx 0.03}$

$0.25 - \frac{2}{3^2} =$

2. (12 points) The correct answer with no supporting work receives *no points*. You must show at least one intermediate step. You do not have to simplify your answers.

(a) (4 points) If $f(x) = \dfrac{\sqrt{x} + 2x^4}{3x}$, find $f'(x)$.

$$f(x) = \frac{x^{1/2}}{3x} + \frac{2x^4}{3x}$$

$$= \frac{1}{3} x^{-1/2} + \frac{2}{3} x^3$$

$$\boxed{f'(x) = \frac{1}{3} \cdot \frac{-1}{2} x^{-3/2} + \frac{2}{3} 3x^2}$$

$f'(x) =$ $-\frac{1}{6} x^{-3/2} + 2x^2$

(b) (4 points) If $y = x^2\left(\dfrac{5}{x} + x^6\right)$, find $\dfrac{dy}{dx}$.

$$y = 5x + x^8$$

$$\frac{dy}{dx} = 5 + 8x^7$$

$\dfrac{dy}{dx} =$ $5 + 8x^7$

(d) (4 points) If $g(x) = \sqrt[3]{x} - 2x^4 + \dfrac{13}{\sqrt{x}} + 7$, find the slope of the tangent line to $g(x)$ at $x = 1$.

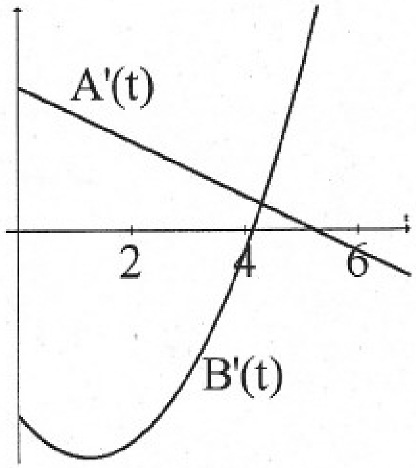
$$g(x) = x^{1/3} - 2x^4 + 13x^{-1/2} + 7$$

$$g'(x) = \frac{1}{3} x^{-2/3} - 8x^3 - \frac{13}{2} x^{-3/2}$$

$$g'(1) = \frac{1}{3} - 8 - \frac{13}{2} = -14.1\overline{6}$$

slope = $\boxed{g'(1) = -14.16}$

3. (14 points) The *rate of ascent* graphs for two balloons (A and B) are given at the right. You are given the ***altitude formula*** *for Balloon B*:



$$B(t) = 0.2t^3 - 0.5t^2 - 2.7t + 90$$

In addition, you are given the ***rate of ascent formula*** *for Balloon A*:

$$A'(t) = -0.4t + 2.1$$

(a) (3 points) Find all times, t, when the *altitude* graph for Balloon A has a horizontal tangent.

$-0.4t+2.1=0 \Rightarrow t=\frac{-2.1}{-0.4}$

$\boxed{t = 5.25}$

(b) (3 points) Which of the following best describes the *altitude* of Balloon B from $t = 0$ to $t = 3$?

i. It increases and then decreases.

ii. It always decreases.

iii. It always increases.

iv. It decreases and then increases.

rate of ascent always negative

ANSWER: ii

(c) (4 points) Find all times after t=0 when Balloon A and Balloon B have the same rate of ascent?

$B'(t) = A'(t)$

$0.6t^2 - t - 2.7 = -0.4t + 2.1$

$0.6t^2 - 0.6t - 4.8 = 0$

$t = \frac{0.6 \pm \sqrt{(0.6)^2 - 4(0.6)(-4.8)}}{2(0.6)}$

$t = 3.37228\ldots$ or -2.37228

$\boxed{t = 3.37}$

(d) (4 points) Find all times, t, when the *rate of ascent* graph for Balloon B has a horizontal tangent.

$B'(t) = 0.6t^2 - t - 2.7$

Has a horizontal tangent at the vertex.

vertex at $t = \frac{-(-1)}{2(0.6)}$

Or use calculus $1.2t - 1 = 0 \Rightarrow t = \frac{1}{1.2} = 0.8\bar{3}$

$t = 0.8\bar{3}$

≈ 0.83

4. (12 points) You are given the total revenue function

$$R(q) = q^2 - 3$$

You do not know the total cost function, but you are given the formula for the slope of secant to the cost graph:

$$\frac{C(q_2) - C(q_1)}{q_2 - q_1} = 4q_2 + q_1 + 11$$

(a) (~~3~~ 4 points) Determine the value of $C(3.01) - C(3)$.

Let $q_1 = 3$ and $q_2 = 3.01$

$$\frac{C(3.01) - C(3)}{3.01 - 3} = 4(3.01) + 3 + 11 = 26.04$$

$$\frac{C(3.01) - C(3)}{0.01} = 26.04$$

→ multiply both sides by 0.01

$C(3.01) - C(3) =$ $\boxed{0.2604}$

(b) (~~3~~ 4 points) Expand the expression $\dfrac{R(4+h) - R(4)}{h}$.

Simplify as much as possible.

$$\frac{R(4+h) - R(4)}{h} = \frac{[(4+h)^2 - 3] - [4^2 - 3]}{h}$$

$$= \frac{16 + 8h + h^2 - \cancel{3} - \cancel{16} + \cancel{3}}{h}$$

$$= \frac{8h + h^2}{h} = \frac{8h}{h} + \frac{h^2}{h} = 8 + h$$

$\dfrac{R(4+h) - R(4)}{h} =$ $\boxed{8+h}$

(~~b~~ c) (~~3~~ 4 points) Find a formula for the derivative of $C(q)$.

Let $q_1 = q$ and $q_2 = q + h$.

$$\frac{C(q+h) - C(q)}{h} = 4(q+h) + q + 11 = 5q + 4h + 11$$

Now let $h \to 0$

$C'(q) =$ $\boxed{5q + 11}$