Nichifor, Win. 2010, Midterm 1, version 1

1 (7 points) The function $g(x)$ is given by: $g(x) = 2x^2 - x$.

a) (5 pts) Write a formula in terms of m and h for the **slope of the secant line** through the graph of $g(x)$ at the points $x = m$ and $x = m + h$. Show work and simplify your answer as far as possible.

$$\frac{g(m+h) - g(m)}{h} = \frac{[2(m+h)^2 - (m+h)] - [2m^2 - m]}{h}$$
$$= \frac{2(m^2 + 2mh + h^2) - m - h - 2m^2 + m}{h}$$
$$= \frac{\cancel{2m^2} + 4mh + 2h^2 - \cancel{m} - h - \cancel{2m^2} + \cancel{m}}{h}$$
$$= \frac{4mh + 2h^2 - h}{h} = \frac{(4m + 2h - 1)h}{h} = 4m + 2h - 1$$

Answer $\boxed{4m + 2h - 1}$:

b) (2 pts) Use the answer you obtained in part (a) to compute $g'(m)$. Clearly indicate your steps.

Let $h \to 0$, then the slope of the secant line $4m + 2h - 1$ approaches $4m - 1$

Answer: $g'(m) = \boxed{4m - 1}$

2 (8 points) Show work and simplify your answers.

a) Find the derivative $f'(x)$ of the function $f(x) = (1 - x^2)(3 + 5x)$.

$$f(x) = (1 - x^2)(3 + 5x) = 3 + 5x - 3x^2 - 5x^3$$
$$f'(x) = 0 + 5 - 3(2x) - 5(3x^2)$$

Answer: $f'(x) = \boxed{5 - 6x - 15x^2}$

b) Find $\frac{ds}{dt}$, if $s = 2\sqrt[3]{t^2} - \frac{5}{\sqrt{t}} + 3$.

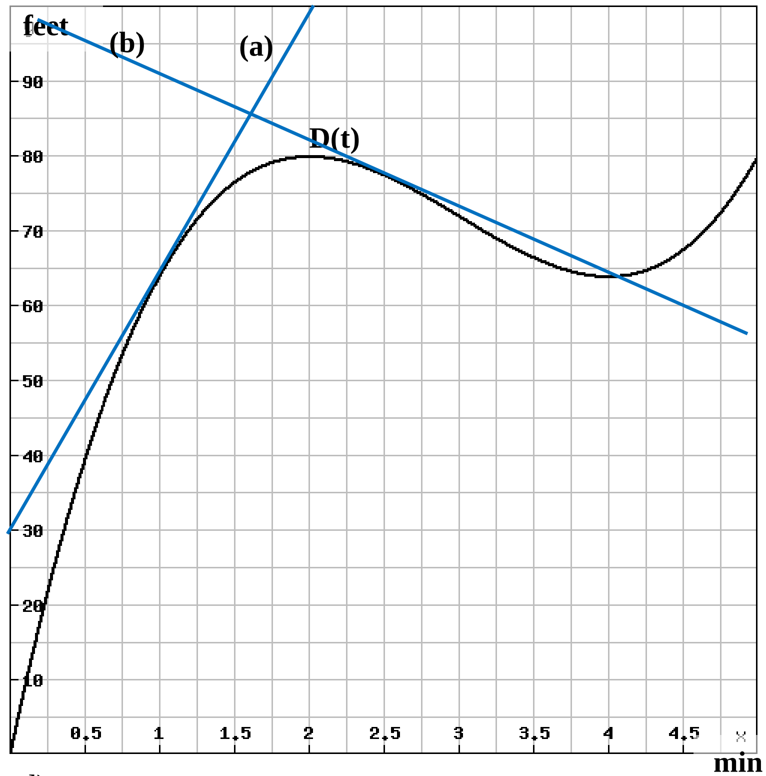
$$s = 2\sqrt[3]{t^2} - \frac{5}{\sqrt{t}} + 3 = 2t^{2/3} - 5t^{-1/2} + 3$$

$$\frac{ds}{dt} = 2\left(\frac{2}{3}t^{\frac{2}{3}-1}\right) - 5\left(-\frac{1}{2}t^{-\frac{1}{2}-1}\right) + 0 = \frac{4}{3}t^{-1/3} + \frac{5}{2}t^{-3/2}$$

Answer: $\frac{ds}{dt} = \boxed{\frac{4}{3}t^{-1/3} + \frac{5}{2}t^{-3/2}}$ or $\boxed{\frac{4}{3\sqrt[3]{t}} + \frac{5}{2\sqrt{t^3}}}$

Nichifor, Win. 2010, Midterm 1, version 1

3 (10 points) The following is the graph of a function $D(t)$, representing the distance along a straight road between a remote-controlled car and its owner.



a) (3 pts) Estimate carefully the instantaneous speed of the car at 1 minute. Show your work.

$$\frac{65-30}{1-0}=35$$

Answer: $\boxed{\cong 35}$ ft/min

(a reasonable range of values was accepted)

b) (3 pts) Estimate the value of $\frac{D(2.6)-D(2.5)}{0.1}$. Show your work.

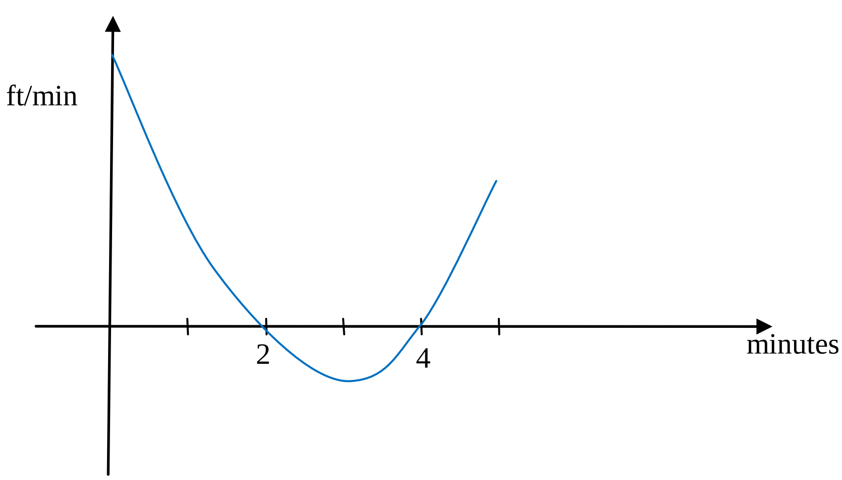
$\frac{D(2.6)-D(2.5)}{0.1}=$ slope of secant line from 2.5 to 2.6

$\approx$ slope of tangent line at 2.5 $\approx \frac{60-95}{4.5-0.5}=\frac{-35}{4}=-8.75$

(a reasonable range of values was accepted)

Answer: ≈ -8.75

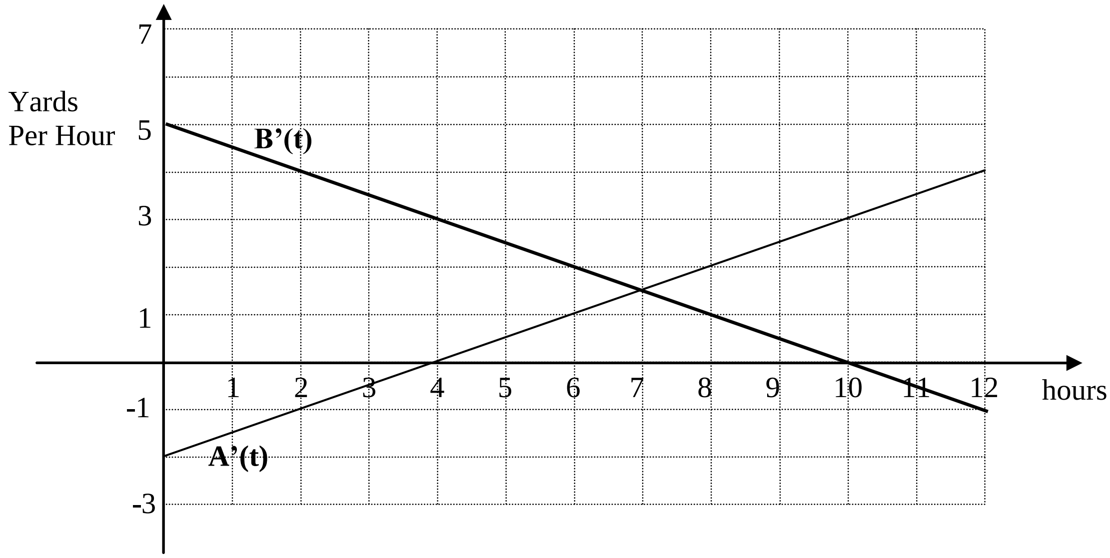
c) (4 pts) Sketch a rough graph of the **speed** of this car. Label the x-intercepts. No need to label any y-coordinates.



Nichifor, Win. 2010, Midterm 1, version 1

4 (12 Points) In this problem you don't need to justify any of your answers.

Two balloons, A & B, start off at the same altitude of 40 yards above ground at time $t = 0$, then they move up and down. The graphs below are the corresponding **rate-of-ascent graphs** for the two balloons.



a) For each of the following statements **circle the correct answer**: True (T), False (F), or cannot tell based on the given information (CT).

i)	At $t = 2$ hours, balloon A's altitude is higher than 40 yards.	**T**	**F**	**CT**
ii)	At time $t = 7$ hrs, balloon B is higher than balloon A.	**T**	**F**	**CT**
iii)	The distance between the two balloons is greater at $t = 2$ than at $t = 4$	**T**	**F**	**CT**

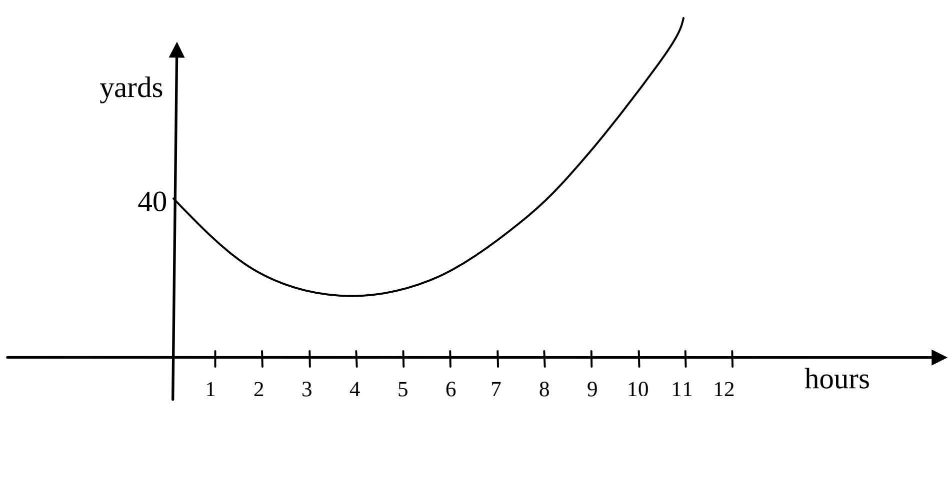
b) Find a 2 hour time interval, if any exists, during which balloon A is ascending but balloon B is descending**.**

Answer: From $t =$__10____ to $t =$___12____ hours, --OR-- circle "none exists".

c) When is the distance between the two balloons the greatest?

Answer: At $t =$__7__ hours.

d) Sketch $A(t)$, the **altitude graph** for balloon A. Label the y-intercept (no other y-coordinates are required)

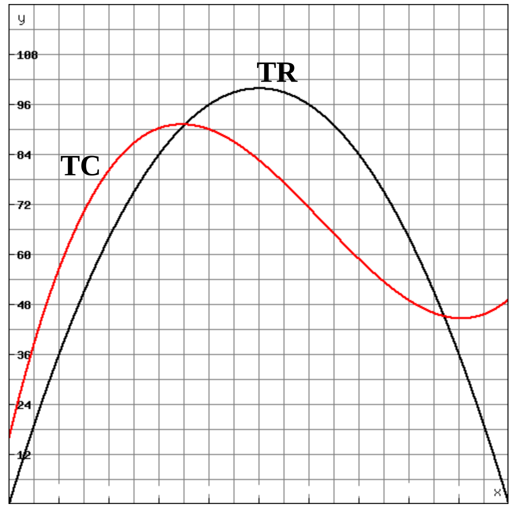


Nichifor, Win. 2010, Midterm 1, version 1

5 (13 Points) The Total Revenue and Total Cost, in **hundreds of dollars**, for producing q **hundred Items** are given by the following formulas:

$$TR(q) = -q^2 + 20q$$

$$TC(q) = \frac{1}{15}q^3 - \frac{25}{10}q^2 + 25q + 16$$



a) (5 pts) Use the derivative rules to write formulas in terms of q for the Marginal Revenue and the Marginal Cost at q hundred Items.

$MR(q) = -2q + 20$

$MC(q) = \frac{1}{5}q^2 - 5q + 25$ *Units for both MR and MC:* *$ per Item (or $)*

b) (5 pts) Compute the quantity that maximizes the profit. Round your answer to the nearest two decimal digits.

Set $MR = MC$ and solve for q, then interpret your answers.

$$-2q + 20 = \frac{1}{5}q^2 - 5q + 25$$

$$\frac{1}{5}q^2 - 3q + 5 = 0$$

Quadratic Formula $MR = MC$ at: $q \approx 1.90983$ ... and $q \approx 13.09016$ ...

From the provided graphs we see that the larger quantity corresponds to max profit (while lower quantity results in max loss).

Answer: Profit is maximal at ___13.09___ hundred Items

c) (3 pts) Compute the maximum possible profit. Round your answer to the nearest two decimal digits.

$$TR(13.09) = -(13.09)^2 + 20(13.09) = 90.4519$$

$$TC(13.09) = \frac{1}{15}(13.09)^3 - \frac{25}{10}(13.09)^2 + 25(13.09) + 16 = 64.409525 \ldots$$

$$P(13.09) = TR(13.09) - TC(13.09) = 90.4519 - 64.409525 \ldots = 26.0423747333333 \approx 26.04$$

Or:

$$P(q) = TR(q) - TC(q) = -\frac{1}{15}q^3 + \frac{15}{10}q^2 - 5q - 16,$$

$$P(13.09) = -\frac{1}{15}(13.09)^3 + \frac{15}{10}(13.09)^2 - 5(13.09) - 16 \approx 26.04$$

Answer: The maximum profit is__26.04_____ hundred dollars