MATH 112 A

Exam II - Version 1

May 22, 2003

Name ______________________________

Student ID #______________

Section ______________

1	17	
2	16	
3	17	
Total	50	

- Check that your exam contains three questions.
- You are allowed to use a calculator, a ruler, and one sheet of handwritten notes.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit.
- Write your answers in the specified locations.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so. If you still need more paper, please ask for some.
- Raise your hand if you have a question.
- Put your name on your sheet of notes and turn it in with the exam.
- You have 50 minutes to complete the exam.

GOOD LUCK!

1. (17 points) The value of a stock you're keeping your eye on appears to grow exponentially. You wish to find an exponential model to predict $V(t)$, the value of one share of the stock in dollars, t days after your observation begins. As a good Math 112 student, you know that the way to do this is to take natural logarithms and find $z = mt + b$, the line of best fit for the logarithmic data. You compute the mean-squared error function for the logarithmic data:

$$E(b, m) = b^2 + 7.5m^2 + 5bm - 0.3b - 0.875m + 0.02585.$$

   (a) (6 points) Compute the partial derivatives $\dfrac{\partial E}{\partial b}$ and $\dfrac{\partial E}{\partial m}$.

   ANSWERS: $\dfrac{\partial E}{\partial b}$ = ____________________

   $\dfrac{\partial E}{\partial m}$ = ____________________

   (b) (5 points) Find the line of best-fit: $z = mt + b$.

   ANSWER: ____________________

   (c) (2 points) What is the smallest possible value of $E(b, m)$? (Do not round your answer.)

   ANSWER: ____________________

   (d) (4 points) Compute the value of your stock in dollars, 6 days after your observation begins. (Round to the nearest cent.)

   ANSWER: $____________________

2. (16 points)

(a) (4 points) Compute the partial derivative $\dfrac{\partial z}{\partial t}$ if $z = t^2 e^m + \dfrac{m^2 + 2m}{\sqrt{t}}$.

ANSWER: $\dfrac{\partial z}{\partial t} =$ ____________________

(b) (6 points) Let $g(x) = \dfrac{1}{5}x^2 - 4x + 32$ and define a new function $S(x)$ as the slope of the diagonal to the point $(x, g(x))$. So, $S(x) = \dfrac{g(x)}{x}$. Find all positive values of x at which $S(x)$ has a horizontal tangent.

ANSWER: $x =$ ____________________

(c) (6 points) Let $k(x) = \dfrac{1}{4}x^4 - \dfrac{11}{3}x^3 + 20x^2 - 48x + 15$. Use the Second Derivative Test to determine whether $k(x)$ has a local maximum, a local minimum, or neither at $x = 3$. If the Second Derivative Test is inconclusive, state why.

ANSWER: (circle one) maximum minimum neither inconclusive

3. (17 points) Mike has a business selling cookies and cupcakes. Each batch of baked goods must go through two stations in Mike's kitchen: the baking station and the packaging station. Each station can only be used on one item (either cookies or cupcakes) at a time. Each batch of cookies takes 40 minutes in the preparation station and 30 minutes in the packaging station. Each batch of cupcakes takes 80 minutes in the preparation station and 20 minutes in the packaging station. The equipment in the baking station can be used for no more than 8 hours (480 minutes) a day. The equipment in the packaging station can be used for no more than 4 hours (240 minutes) a day. A batch of cookies brings in $25.80 in profit. A batch of cupcakes brings in $21.50 in profit.

   Suppose Mike makes and sells x batches of cookies and y batches of cupcakes in a day. He wishes to maximize his daily profit.

   (a) (6 points) Find the objective function and the constraints for this problem.

   ANSWER: objective function: ____________________

   constraints: ____________________

   ____

   (b) (6 points) Sketch the feasible region, clearly labelling all vertices.

   (c) (3 points) Determine how many batches of each baked item Mike should sell in order to maximize profit.

   ANSWER: cookies: ______batches, cupcakes: ______batches

   (d) (2 points) What is Mike's maximum possible profit?

   ANSWER: $__________________