MATH 112
Exam II - Version 1
May 17, 2005

Name _____

Student ID #_____ Section_____

HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

SIGNATURE: _____

1	16	
2	12	
3	14	
4	8	
Total	50	

- Please check that your exam contains four problems.
- Please turn your cell phone OFF and put it away for the duration of the exam.
- Unless otherwise indicated, you must show your work. The correct answer with no supporting work may result in no credit.
- If you use a guess-and-check method when an algebraic method is available, you may not receive full credit.
- Put your name on your sheet of notes and turn it in with the exam.

GOOD LUCK!

1. (16 points) Find the indicated derivative. Do not simplify.

   (a) $y = \ln(\sqrt[3]{x} + e^{4x^5})$

   $\dfrac{dy}{dx} =$

   (b) $G(t) = \left[\dfrac{x^3}{1 + e^{-8x}}\right]^{10}$

   $G'(t) =$

   (c) $z = \dfrac{1}{\sqrt{x^2 + (4y + 3)^2}}$

   $\dfrac{\partial z}{\partial y} =$

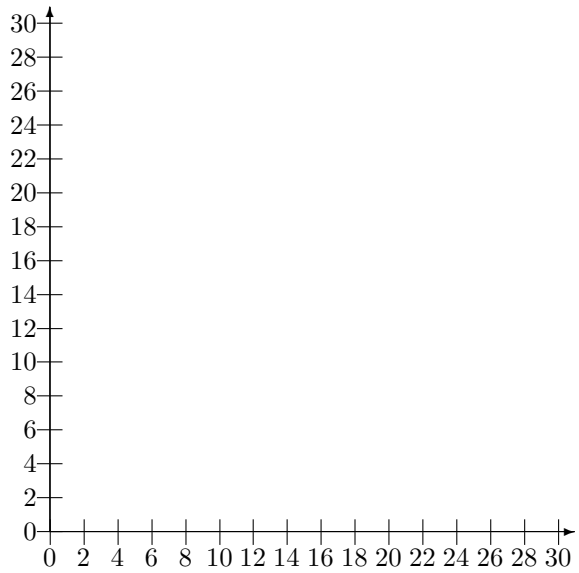
   (d) $f(x, y) = x^2 e^x y$

   $f_x(x, y) =$

2. (12 points) The constraints for a linear programming problem are:

$$5x + 10y \leq 133,\ x \leq 20,\ \text{and } y \leq 8.$$

(a) Sketch the feasible region.



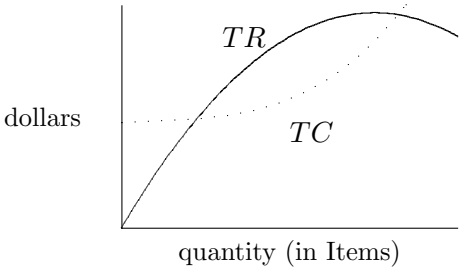
(b) Find the exact coordinates of the vertices of the feasible region. Label all of them on your graph.

(c) Find the maximum and minimum values of the objective function $f(x, y) = 0.10x + 0.25y$, subject to these constraints.

ANSWER: minimum=_____; maximum=_____

3. (14 points) You sell Items. The graphs of total revenue and total cost are given at right. Their formulas are:

$$TR(q) = -3q^2 + 90q \text{ and}$$
$$TC(q) = 0.1q^3 - 0.6q^2 + 3.6q + 330.$$



(a) Find the positive value of q at which the graph of profit has a horizontal tangent line. Use the Second Derivative Test to show that profit is maximized at this quantity.

ANSWER: $q =$_____Items

(b) Variable cost is given by the function $VC(q) = 0.1q^3 - 0.6q^2 + 3.6q$ and average variable cost is given by $AVC(q) = \dfrac{VC(q)}{q}$. Give the largest and smallest values of AVC on the interval from $q = 1$ to $q = 10$.

ANSWER: smallest = \$_____per Item;
largest = \$_____per Item

(c) Is the graph of marginal cost concave up or concave down at $q = 1$? Show work to justify your answer.

ANSWER: (circle one) concave up concave down

4. (8 points) The mean-squared error function for a set of data is given by the function $E(b, m)$. The partial derivatives of this function are:

$$\frac{\partial E}{\partial b} = 2b + 15m - 26 \text{ and } \frac{\partial E}{\partial m} = 142m + 15b - 240.$$

(a) Find the line $y = mx + b$ that has the smallest possible mean-squared error for this set of data.

ANSWER: $y =$_____

(b) Approximate $\dfrac{E(1, 2.003) - E(1, 2)}{0.003}$.

ANSWER: $\dfrac{E(1, 2.003) - E(1, 2)}{0.003} \approx$_____

(c) Find a value of b and m at which the following occurs: If you fix m and increase b slightly, then E increases. Justify your answer.

ANSWER: $b =$_____; $m =$_____