

MATH 112
Exam II
May 18, 2006

Name _____

Student ID # _____

Section _____

HONOR STATEMENT

“I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam.”

SIGNATURE: _____

1	17	
2	16	
3	17	
Total	50	

- Your exam should consist of this cover sheet, followed by 3 problems on 5 pages. Check that you have a complete exam.
- Turn your cell phone OFF and put it away for the duration of the exam.
- Unless otherwise indicated, you must show your work. The correct answer with no supporting work may result in no credit.
- If you use a guess-and-check method when an algebraic method is available, you may not receive full credit.
- Put your name on your sheet of notes and turn it in with the exam.

GOOD LUCK!

1. (17 points)

(a) Let $z = \frac{x^4 y}{x^2 + 3y}$. Compute $\frac{\partial z}{\partial x}$. Do not simplify.

(b) Let $w = (t \ln(r) + 1) \cdot e^{tr}$. Compute $\frac{\partial w}{\partial t}$. Do not simplify.

- (c) Kris sells radishes and sparkplugs. Kris' profit (in thousands of dollars) on an order of r thousand radishes and s thousand sparkplugs is

$$P(r, s) = 340 + 5rs + \frac{30}{r} + \frac{25}{s}.$$

- i. Compute $P_r(r, s)$ and $P_s(r, s)$.

ANSWER: $P_r(r, s) =$ _____

ANSWER: $P_s(r, s) =$ _____

- ii. If Kris has sold $r = 2$ thousand radishes and $s = 4$ thousand sparkplugs, which will lead to the greater increase in profit: selling one more radish or selling one more sparkplug? (Use partial derivatives to answer the question and show your work.)

ANSWER: selling one more (circle one) radish sparkplug

- (d) We do not know the formula for a function $g(x, y)$, but we do know its partial derivatives:

$$g_x(x, y) = \frac{x^2 - 2xy - 3y}{(x - y)^2} \text{ and } g_y(x, y) = \frac{x^2 + 3x - 6y}{(x - y)^2}.$$

Approximate the value of

$$\frac{g(2, 3.0001) - g(2, 3)}{0.0001}.$$

ANSWER: $\frac{g(2, 3.0001) - g(2, 3)}{0.0001} \approx$ _____

2. (16 points) A pharmaceutical company produces two drugs: Xemprex and Yozak. Each drug is processed on two machines: the mixer and the coater. Each pound of Xemprex requires 0.2 hours in the mixer and 0.1 hours in the coater; while each pound of Yozak requires 0.1 hours in the mixer and 0.1 hours in the coater. Each pound of Xemprex yields \$73 profit and each pound of Yozak yields \$36 profit.

Let x be the daily production (in pounds) of Xemprex and y be the daily production (in pounds) of Yozak. Use the method of linear programming to find the values of x and y that maximize profit if production is limited by the following constraints:

- The mixer can be used for no more than 15.1 hours a day.
- The coater can be used for no more than 9.8 hours a day.
- Due to FDA restrictions, the company can produce no more than 60 pounds of Xemprex in a day.

Show all your work.

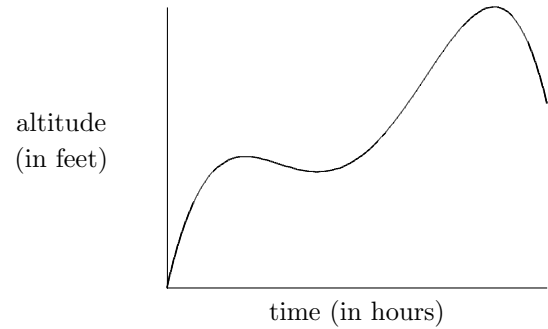
ANSWER: $x =$ _____, $y =$ _____

3. (17 points)

The graph of a hot-air balloon's altitude is given at right. The altitude is given by the function:

$$A(t) = -\frac{1}{3}t^4 + 13t^3 - 160t^2 + 750t,$$

where t is in hours and $A(t)$ is in feet.



- (a) The overall average vertical speed of the balloon is given by $S(t) = \frac{A(t)}{t}$. Write out formulas for $S(t)$ and $S'(t)$.

ANSWER: $S(t) =$ _____

$S'(t) =$ _____

- (b) Find all values at which $S(t)$ has a horizontal tangent.

ANSWER: $t =$ _____

- (c) Find the largest and smallest values of $S(t)$ on the interval from $t = 15$ to $t = 20$.

ANSWER: largest = _____ feet per hour

smallest = _____ feet per hour

- (d) A second balloon has altitude function $B(t)$. You do not know the formula for $B(t)$, but you know the formula for $B'(t)$:

$$B'(t) = \sqrt{t+5} - \left(\frac{1}{2}t + 1\right).$$

Find the time at which $B(t)$ has a horizontal tangent.

ANSWER: $t =$ _____ hours

- (e) Use the Second Derivative Test to determine whether the time you found in part (d) gives a local maximum or a local minimum of $B(t)$.

ANSWER: (circle one) local max local min