

NAME: \_\_\_\_\_

Student ID #: \_\_\_\_\_

QUIZ SECTION: \_\_\_\_\_

Version 1

**Math 112 B**  
**Midterm II**  
May 16, 2006

Problem 1	18	
Problem 2	16	
Problem 3	16	
<b>Total:</b>	<b>50</b>	

- You are allowed to use a calculator, a ruler, and one sheet of notes.
- Your exam should contain 4 pages in total and 3 problems.  
Make sure you have a complete test.
- Unless otherwise noted, you **must show how you get your answers**.  
Correct (or incorrect) answers with no supporting work may result in little or no credit.
- If an algebraic method is available, answers obtained by guessing, approximating, using your graphing calculator, or plug-and-check will get little or no credit.
- Write your **final answer in the indicated spaces**. Unless otherwise noted, round your answer to two decimal digits.
- If you need more room, use the backs of pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

GOOD LUCK!

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*Do you want me to post your grade so far on the class website under the last 4 digits of your STUDENT ID (in about a week)?*

*Yes, please post my grade. Sign to give permission: \_\_\_\_\_*

*No, please don't post my grade so far.*

1) (18 points) Compute the indicated derivatives of the following functions. **Do not simplify.**

a)  $f(x) = xe^x \sqrt{x+1}$  Product Rule 2 times

$$f'(x) = (e^x + xe^x) \sqrt{x+1} + (xe^x) \frac{1}{2} (x+1)^{-1/2}$$

b)  $h(z) = e^{z^2+z}$  Chain Rule (Gen. Exp. Rule)

$$h'(z) = e^{z^2+z} (2z+1)$$

c)  $y = \left(\frac{\ln t}{t^3}\right)^5$  Chain (Power) + Quotient

$$\frac{dy}{dt} = 5 \left(\frac{\ln t}{t^3}\right)^4 \left(\frac{\frac{1}{t} \cdot t^3 - \ln t (3t^2)}{(t^3)^2}\right)$$

d)  $g(x, y) = x^3 + \frac{1}{2}xy^2 - \frac{8x}{y} + 7$

i)  $g_x(x, y) = 3x^2 + \frac{1}{2}y^2 - \frac{8}{y}$

ii)  $g_y(x, y) = \frac{1}{2}x(2y) - 8x(-y^{-2}) = xy + \frac{8x}{y^2}$

2. (16 points) Your Total Revenue (in dollars) vs. the quantity  $q$  of Items sold is given by the function:

$$TR(q) = 0.03q^4 - 1.5q^3 + 20q^2 + q.$$

Recall that the Average Revenue is given by  $AR(q) = \frac{TR(q)}{q}$  dollars per Item.

a) Write out formulas in terms  $q$  for  $AR(q)$  and its derivative,  $AR'(q)$ . Simplify as much as possible.

$$AR(q) = 0.03q^3 - 1.5q^2 + 20q + 1$$

$$AR'(q) = 0.09q^2 - 3q + 20$$

b) Find the critical points for the function  $AR(q)$ .

$$AR'(q) = 0 : 0.09q^2 - 3q + 20 = 0$$

Quadratic Formula gives:

ANSWER: critical points are at  $q = 9.21 \text{ \& } 24.12$ .

c) What is the maximum value of the Average Revenue function on the interval  $q=10$  to  $q=30$  Items?

Compute AR at critical points within the interval (i.e. 24.12) and at endpoints (10 & 30) then pick the max

$$AR(10) = 0.03(10)^3 - 1.5(10)^2 + 20(10) + 1 = 81$$

$$AR(24.12) \approx 31.71$$

$$AR(30) = 61$$

ANSWER: Maximum AR for  $q$  between 10 and 30 Items is 81 \$/Item

d) Determine whether the Average Revenue function  $AR(q)$  is increasing or decreasing at  $q=20$  Items.

$$AR'(20) = 0.09(20)^2 - 3(20) + 20 = -4 < 0$$

So AR is decreasing

ANSWER (circle one):  $AR(q)$  is increasing / decreasing at  $q=20$ .

e) Determine whether the graph of  $AR(q)$  is concave up or down at  $q=20$  Items.

$$AR''(q) = 0.18q - 3$$

$$AR''(20) = 0.18(20) - 3 = 0.6 > 0 \text{ So AR is concave up}$$

ANSWER (circle one):  $AR(q)$  is concave up / concave down at  $q=20$

3. (16 points) You want to buy pizza for a party you are planning. There are two types of pizzas your friends like: VeggieHeaven and MeatGalore. The VeggieHeaven pizza costs \$8 and feeds 5 people. The MeatGalore costs \$10 and feeds 6.

You have \$110 to spend, and the pizza place says they only have 10 VeggieHeaven pizzas left so you can buy at most that many.

Let  $x$  denote the number of VeggieHeaven and let  $y$  denote the number of MeatGalore pizzas you buy.

You want to maximize the number of friends  $F(x,y)$  you can feed, subject to the given constraints.

a) Find the formula for your objective function  $F(x,y)$

$$F(x,y) = 5x + 6y$$

b) Write the inequalities for your constraints:

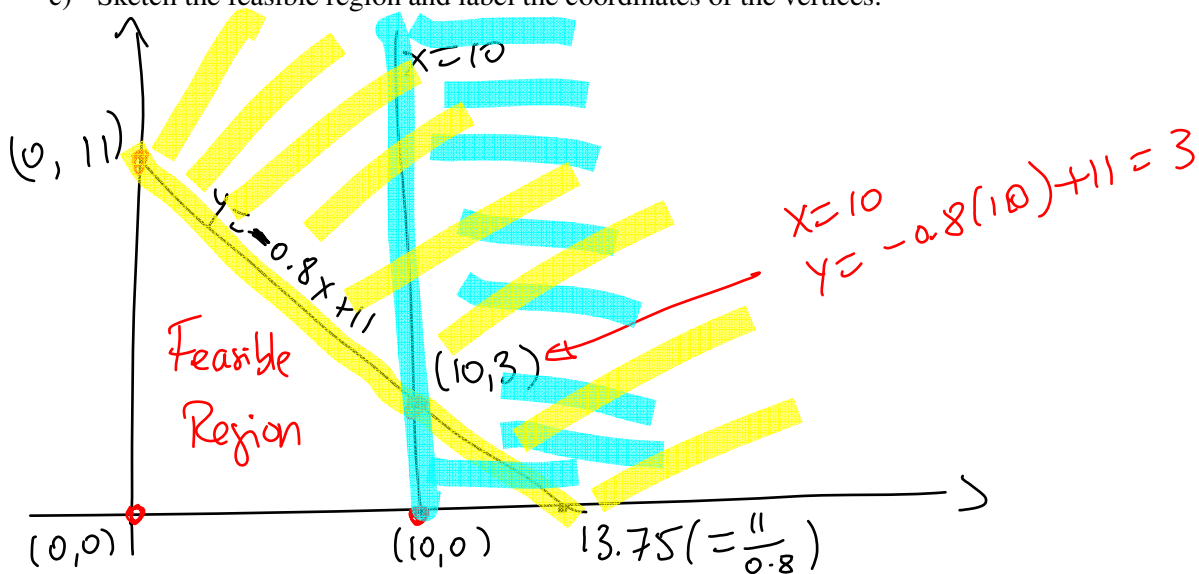
$$8x + 10y \leq 110$$

$$x \leq 10$$

( Lines used below )

$$\begin{aligned} &\rightsquigarrow y = -0.8x + 11 \\ &\rightsquigarrow x = 10 \end{aligned}$$

c) Sketch the feasible region and label the coordinates of the vertices.



d) Find the maximum number of friends you can invite to the party and how many pizzas of each kind you should order.

$$F(10,0) = 0$$

$$F(0,11) = 5 \cdot 0 + 6 \cdot 11 = 66$$

$$F(10,0) = 5 \cdot 10 + 6 \cdot 0 = 50$$

$$\boxed{F(10,3) = 5 \cdot 10 + 6 \cdot 3 = 68}$$

max

ANSWER: Can invite up to 68 friends and should order  $x = \underline{10}$  VH's and  $y = \underline{3}$  MG's