

1. (12 pts) Compute the indicated derivatives and put a box around your final answer. Do not simplify.

(a) (4 pts)  $G(w, b) = b^3 m + 4 \ln(m) - \frac{5}{b^2} - m^3 b^6 + 7m + 14$

$$\frac{\partial G}{\partial b} = 3b^2 m + 10b^{-3} - 6m^3 b^5$$

$$\frac{\partial G}{\partial m} = b^3 + \frac{4}{m} - 3m^2 b^6 + 7$$

(b) (4 pts)  $y = \ln(e^{3x^5} + \sqrt{3x-2})$

$$\frac{dy}{dx} = \frac{1}{e^{3x^5} + \sqrt{3x-2}} \left( e^{3x^5} \cdot 15x^4 + \frac{1}{2}(3x-2)^{-\frac{1}{2}} \cdot 3 \right)$$

(c) (4 pts)  $F(x) = \frac{(x^2+1)^{12}}{1+\ln(2x+1)}$

$$F'(x) = \frac{(1+\ln(2x+1)) 12(x^2+1)^{11} 2x - (x^2+1)^{12} \frac{1}{2x+1} \cdot 2}{(1+\ln(2x+1))^2}$$

2. (12 pts) Let  $f(x, y) = 4y^2 + 3xy - 24x + 10$ .

(a) (4 pts) Compute the partial derivatives  $f_x(x, y)$  and  $f_y(x, y)$ .

ANSWERS:

$$f_x(x, y) = 3y - 24$$

$$f_y(x, y) = 8y + 3x$$

(b) (4 pts) If we fix  $y$  to be 3, then  $g(x) = f(x, 3)$  becomes a function of only one variable, the variable  $x$ . Find the slope of the tangent line to the one variable function  $g(x)$  when  $x = 5$  and circle the partial derivative expression that corresponds to this value.

$$\begin{aligned} g(x) &= 4(3)^2 + 3x \cdot 3 - 24x + 10 \\ &= -15x + 46 \end{aligned}$$

$$g'(x) = -15 \quad g'(5) = -15$$

$$f_x(5, 3) = -15$$

ANSWER: slope = -15

ANSWER (circle one):  $f_x(5, 3)$   $f_x(5, 5)$   $f_y(5, 3)$   $f_y(5, 5)$

(c) (4 pts) Find all pts  $(x, y)$  that are candidates for a local maximum or local minimum of  $f(x, y)$ .

$$3y - 24 = 0$$

$$3y = 24$$

$$y = 8$$

$$8y + 3x = 0$$

$$64 + 3x = 0$$

$$3x = -64$$

$$x = -\frac{64}{3}$$

ANSWER: list of candidates  $(x, y) = \underline{\left(-\frac{64}{3}, 8\right)}$

3. (14 pts) Your Total Cost (in dollars) vs. the quantity  $q$  of Items sold is given by the function:

$$TC(q) = \frac{q^3}{3} - 5q^2 + 21q + \frac{55}{2}$$

The Average Cost is given by  $AC(q) = \frac{TC(q)}{q}$ .

- (a) (6 pts) Find the two values of  $x$  at which the Total Cost graph has a horizontal tangent. Use the Second Derivative Test to determine whether  $TC(q)$  reaches a local maximum or a local minimum at each value.

$$TC'(q) = q^2 - 10q + 21 \stackrel{?}{=} 0$$

$$(q-3)(q-7) = 0 \Rightarrow q = 3, 7$$

$$TC''(q) = 2q - 10$$

$$q = 3 : TC''(3) = 2(3) - 10 = -4 < 0$$

CONCAVE DOWN  $\cap$

$$q = 7 : TC''(7) = 2(7) - 10 = 4 > 0$$

CONCAVE UP  $\cup$

ANSWER:  $q = \underline{3}$  gives a local MAX of  $TC(q)$ ;  
 $q = \underline{7}$  gives a local MIN of  $TC(q)$ .

- (b) (4 pts) Give the global maximum and global minimum values of Total Cost on the interval from  $q = 0$  to  $q = 5$ .

$$TC(0) = \frac{55}{2} = 27.5$$

$$TC(3) = \frac{(3)^3}{3} - 5(3)^2 + 21(3) + \frac{55}{2} = 54.5$$

$$TC(5) = \frac{(5)^3}{3} - 5(5)^2 + 21(5) + \frac{55}{2} = 49.1\bar{6}$$

ANSWER: MAX = \$54.50 dollars; MIN = \$27.50 dollars.

- (c) (4 pts) Is the Average Cost graph concave up, concave down, or neither at  $q = 3$ ? Justify your answer. Guessing the answer with no supporting work receives zero pts.

$$AC(q) = \frac{q^2}{3} - 5q + 21 + \frac{55}{2q}$$

$$AC'(q) = \frac{2}{3}q - 5 - \frac{55}{2}q^{-2}$$

$$AC''(q) = \frac{2}{3} + 55q^{-3}$$

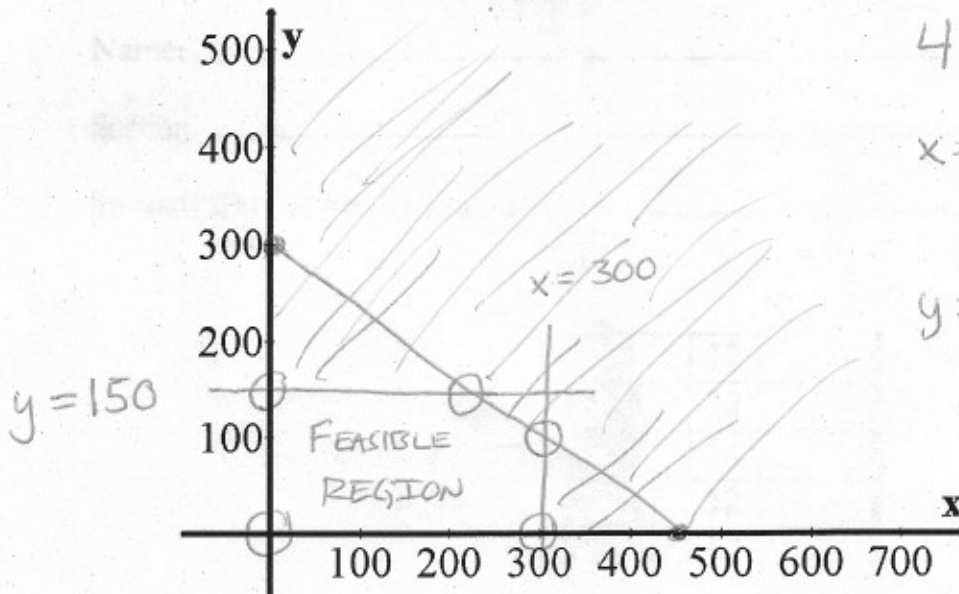
$$AC''(3) = 2.7037$$

ANSWER (circle one): CONCAVE UP    CONCAVE DOWN    NEITHER

4. (12 pts) The constraints for a linear programming problem are

$$4x + 6y \leq 1800, x \leq 300, \text{ and } y \leq 150.$$

(a) (4 pts) Sketch the feasible region.



$$4x + 6y = 1800$$

$$x=0 \Rightarrow 6y = 1800 \\ y = 300$$

$$y=0 \Rightarrow 4x = 1800$$

$$x = 450$$

(b) (4 pts) Find the exact coordinates of the vertices of the feasible region. Label all of them on your graph.

$$(0,0), (300,0), (0,150)$$

INTERSECTIONS

$\boxed{\text{I}} \quad \left. \begin{array}{l} x = 300 \\ 4(300) + 6y = 1800 \\ 6y = 600 \\ y = 100 \end{array} \right\} (300, 100)$	$\boxed{\text{II}} \quad \left. \begin{array}{l} y = 150 \\ 4x + 6(150) = 1800 \\ 4x = 900 \\ x = 225 \end{array} \right\} (225, 150)$
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(c) (4 pts) Subject to the given constraints, find the maximum value of the objective function:

$$f(x, y) = 14x + 20y.$$

$$f(0,0) = 0$$

$$f(300,0) = 4200$$

$$f(0,150) = 3000$$

$$f(300,100) = 6200$$

$$f(225,150) = 6150$$

ANSWER: maximum = 6200