

MATH 112 – EXAM II Hints and Answers  
Spring 2008

1. (a) i. (6 points) ANSWER:  $A'(t) = 7(6t^3 + \ln t)^6 \left(18t^2 + \frac{1}{t}\right) + \frac{1 + e^t}{t + e^t}$
- ii. (6 points) ANSWER:  $\frac{dz}{dx} = \frac{\sqrt{x^2 + 3x} \cdot (e^{4x}) \cdot 4 - e^{4x} \cdot \frac{1}{2}(x^2 + 3x)^{-1/2}(2x + 3)}{(\sqrt{x^2 + 3x})^2}$
- (b) (8 points) HINT:  $f_y(x, y) = 3x^4y^2 - 6xy + \frac{12}{y^3} + (e^{x^3-x}) \cdot \frac{1}{y}$ . Compare  $f_y(1, 1)$ ,  $f_y(0, 1)$ , and  $f_y(-1, 1)$ .  
ANSWER:  $f(-1, y)$  has the steepest graph at  $y = 1$ .
2. (a) (5 points) HINT:  $MR(q) = TR'(q) = (q + 25)^{1/2}$  and  $MC(q) = TC'(q) = \frac{1}{6}q + 3$ . Set these equal to each other, square both sides (being careful to FOIL correctly), and solve the resulting equation.  
ANSWER:  $q = 24$
- (b) (6 points) ANSWER:  $P(q) = \frac{2}{3}(q+25)^{3/2} - \frac{250}{3} - \frac{1}{12}q^2 - 3q - 10$ ,  $P'(q) = \sqrt{q+25} - \frac{1}{6}q - 3$ ,  $P''(q) = \frac{1}{2\sqrt{q+25}} - \frac{1}{6}$
- (c) (3 points) ANSWER:  $P'(24) = \sqrt{24+25} - \frac{1}{6}(24) - 3 = 0$ . So,  $P$  has a horizontal tangent line at  $q = 24$ .  $P''(24) = \frac{1}{14} - \frac{1}{6}$ , which is negative. So,  $P$  is concave down at  $q = 24$ . By the Second Derivative Test,  $P$  has a local max at  $q = 24$ .
- (d) (4 points) ANSWER:  $AC(q) = \frac{1}{12}q + 3 + \frac{10}{q}$ ,  $AC'(q) = \frac{1}{12} - \frac{10}{q^2}$ ,  $AC''(q) = \frac{20}{q^3}$ . In particular,  $AC''(50) = \frac{20}{50^3}$ , which is positive. So,  $AC$  is concave up at  $q = 50$ .
3. (12 points) HINT: The vertices of the feasible region are  $(0, 0)$ ,  $(0, 40)$ ,  $(45, 30)$ ,  $(52, 17)$ , and  $(52, 0)$ . Plug each vertex into the objective function and choose the largest value.  
ANSWER: \$345