MATH 112 SPRING 2010 EXAM 2

$\boxed{1}$ (a) $F(x) = \left(\ln\left(1+\frac{3}{x}\right)\right)^{10} = \left(\ln(1+3x^{-1})\right)^{10}$

CHAIN RULE (POWER RULE FOLLOWED BY LOG RULE)

$$\boxed{F'(x) = 10\left(\ln(1+3x^{-1})\right)^{9} \cdot \frac{1}{1+3x^{-1}} \cdot (-3x^{-2})}$$

(b) $y = \sqrt[4]{2x+1} \cdot e^{(2x^4)} = (2x+1)^{1/4} \cdot e^{(2x^4)}$

PRODUCT RULE

$$\boxed{\frac{dy}{dx} = (2x+1)^{1/4} \cdot e^{(2x^4)} \cdot 8x^3 + e^{(2x^4)} \cdot \frac{1}{4}(2x+1)^{-3/4} \cdot 2}$$

(c) $A(x) = \dfrac{3000\left(1+\frac{x}{100}\right)^{12} - 3000}{x}$

QUOTIENT RULE

$$A'(x) = \frac{x\left(3000 \cdot 12\left(1+\frac{x}{100}\right)^{11} \cdot \frac{1}{100}\right) - (1)\left(3000\left(1+\frac{x}{100}\right)^{12} - 3000\right)}{x^2}$$

$$A'(10) = \frac{10\left(3000 \cdot 12\left(1+\frac{10}{100}\right)^{11} \cdot \frac{1}{100}\right) - \left(3000\left(1+\frac{10}{100}\right)^{12} - 3000\right)}{10^2}$$

$$= \frac{10271.22014 - 6415.2861}{100} = \boxed{38.55935}$$

$$= \boxed{38.56}$$

ASIDE: This means that an increase from $x = 10$ percent to $x = 11$ percent interest would result in an approximate increase of \$38.56 in a year.

2 (a) $TR(q) = (24 - 8\sqrt{q})q = 24q - 8q^{3/2}$

GLOBAL MAX/MIN METHOD

STEP 1 $TR'(q) = 24 - 12q^{1/2} \stackrel{?}{=} 0 \Rightarrow -12\sqrt{q} = -24$

$\Rightarrow \sqrt{q} = 2 \Rightarrow q = 4$ ← critical number

STEP 2 $TR(4) = 24(4) - 8(4)^{3/2} = 32$ ← MAX

STEP 3 $TR(2) = 24(2) - 8(2)^{3/2} = 25.372583$ ← MIN

$TR(6) = 24(6) - 8(6)^{3/2} = 26.42449235$

CORRESPONDING PRICES

MIN $TR = pq$ $\quad p \times 2 = 25.3725 \Leftrightarrow$ $\boxed{p = \$12.69}$

MAX $TR = pq$ $\quad p \times 4 = 32 \Leftrightarrow$ $\boxed{p = \$8.00}$

(b) $TC'(q) = \frac{1}{4}q^2 - q + \frac{3}{4} \stackrel{?}{=} 0$

$q^2 - 4q + 3 = 0$) OR QUADRATIC FORMULA

$(q-3)(q-1) = 0$

$q = 1$ or $q = 3$ ← critical numbers

$TC''(q) = \frac{1}{2}q - 1$

$TC''(1) = \frac{1}{2}(1) - 1 = -\frac{1}{2} < 0$ CONCAVE DOWN $\cap$

$TC''(3) = \frac{1}{2}(3) - 1 = \frac{1}{2} > 0$ CONCAVE UP $\cup$

$\boxed{q = 1 \text{ corresponds to a local } \underline{\text{MAX}}}$

$\boxed{q = 3 \text{ corresponds to a local } \underline{\text{MIN}}}$

3 (a) $\boxed{f_x(x,y) = 14 + 6xy}$ $\boxed{f_y(x,y) = -12 + 3x^2}$

(b) $14 + 6xy \stackrel{?}{=} 0$ AND $-12 + 3x^2 \stackrel{?}{=} 0$

$3x^2 = 12$

$x = -2 \Rightarrow 14 - 12y = 0$ $x^2 = 4$

$\Rightarrow y = \frac{14}{12} = \frac{7}{6}$ $x = -2$ or $x = 2$

$x = +2 \Rightarrow 14 + 12y = 0$

$y = -\frac{14}{12} = -7/6$

CRITICAL POINTS $\boxed{(-2, 7/6), (2, -7/6)}$

(c) $f_x(4,0) = 14 + 6(4)(0) = 14$ ← change with respect to x

$f_y(4,0) = -12 + 3(4)^2 = 36$ ← change with respect to y

$f_x(4,0)$ is $\boxed{\text{SMALLER}}$ than $f_y(4,0)$

(d) $g(x) = f(x, -\frac{1}{3}) = 14x + 4 - x^2$

(i) $g'(x) = 14 - 2x = f_x(x, -\frac{1}{3})$

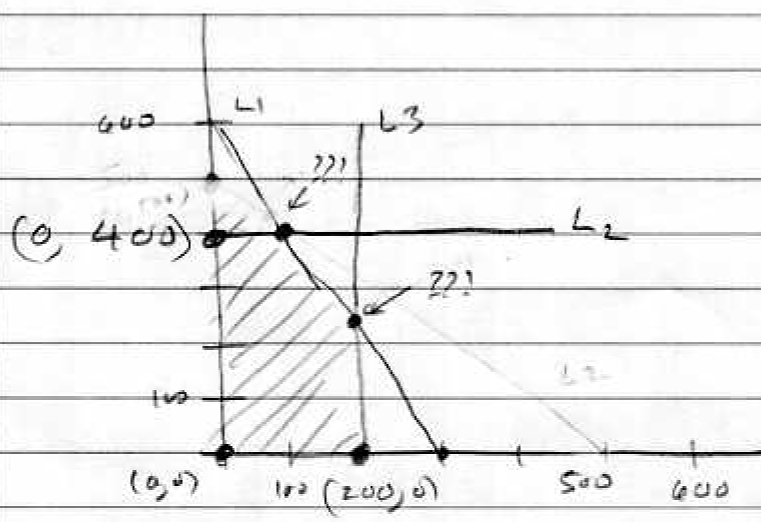
$g'(3) = f_x(3, -\frac{1}{3}) = 14 - 2(3) = 8 > 0$ $\boxed{\text{INCREASING}}$

(ii) $g''(x) = -2 < 0$ $\boxed{\text{CONCAVE DOWN}}$

4 (a) L1 $4x+2y=1200$ goes through the points $(0,600)$ and $(300,0)$

L2 $y=400$ is a horizontal line at 400

L3 $x=200$ is a vertical line at $x=200$.



INTERSECTIONS

L1 AND L2 : $4x+2(400)=1200 \Rightarrow \boxed{x=100}$

$(100,400)$

L1 AND L3 : $4(200)+2y=1200$

$2y=400 \quad \boxed{y=200}$

$(200,200)$

(b) $f(0,0) = 200$

$f(0,400) = 1200+200 = 1400$

$f(200,0) = 400+200 = 600$

$f(100,400) = 200+1200+200 = 1600$

$f(200,200) = 400+600+200 = 1200$

$$\boxed{\begin{array}{l} \text{MIN} = 200 \\ \text{MAX} = 1600 \end{array}}$$