SPRING 2011, Math 112, Midterm II, version 1 Solutions

1 (15 points)

a) $f(x) = \sqrt{x^3e^x + 1} \implies f'(x) = \frac{1}{2}(x^3e^x + 1)^{-\frac{1}{2}}(3x^2e^x + x^3e^x)$

b) $z = \frac{y\ln(y)}{y^2 + 5} \implies \frac{dz}{dy} = \frac{\left(1 \cdot \ln(y) + y \cdot \frac{1}{y}\right)(y^2 + 5) - (y\ln(y))(2y)}{(y^2 + 5)^2}$

c) $g(t) = \ln(\sqrt{t^3 - 2t + 1}) \implies g'(t) = \frac{1}{\sqrt{t^3 - 2t + 1}}\left(\frac{1}{2}(t^3 - 2t + 1)^{-\frac{1}{2}}\right)(3t^2 - 2)$

2 (15 points) Consider the function $\boldsymbol{f(x) = \frac{x^3}{3} - 15x^2 + 200x}$

a) (3 pts) Compute all critical numbers of $f(x)$.

$f'(x) = x^2 - 30x + 200$

Set $f'(x) = 0$

and solve using the Quadratic Formula: $x = \mathbf{10, 20}$ (list all)

b) (5 pts) $f''(x) = 2x - 30$

$f''(10) = -10 < 0$, so, by the second derivative test, $\boxed{x = 10 \textit{ is a local max}}$

$f''(20) = 10 > 0$, so, by the second derivative test, $\boxed{x = 20 \textit{ is a local min}}$

c) (3 pts) The graph of $f(x)$ **Increasing** at $x = 0$, because: $\boldsymbol{f'(0) = 200 > 0}$

d) (4 pts) Determine the **minimum value** of $f(x)$ on the interval from $x = 1$ to $x = 15$. Show work.

Evaluate $f(x)$ at all critical numbers in the given interval ($x = 10$), and the endpoints of the interval

$f(1) = \frac{1}{3} - 15 + 200 \cong \mathbf{185.33}$

$f(10) = \frac{1000}{3} - 15(10)^2 + 200(10) \cong 833.33$

$f(15) = \frac{3375}{3} - 15(15)^2 + 200(15) = 750$

ANSWER: The **minimum value** of $f(x)$ on the given interval is: **185.33**

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3 (10 points)

a) (2pts) $P = 1.7(100)^{0.3}(2500)^{0.8} \cong \mathbf{3,538.35}$

b) (4 pts)

$$\frac{\partial P}{\partial L} = \mathbf{0.51\, L^{-0.7} K^{0.8}}$$

$$\frac{\partial P}{\partial K} = \mathbf{1.36\, L^{0.3} K^{-0.2}}$$

c) (4 pts)

$$\frac{\partial P}{\partial L}(100, 1{,}500) = 0.51\,(100)^{-0.7}(1500)^{0.8} \cong \mathbf{7.05}$$

4 (10 points)

a) (2 pts) Objective: $G(x, y) = \mathbf{0.12x + 0.1y}$

b) (2 pts) Constraints:

Hours: $x + y \leq 16$,

Budget: $30x + 15y \leq \$300$

c) (4pts) Sketch the feasible region and list the coordinates of all the vertices.

$x + y = 16$ *has intercepts* $(0, 16)\&\ (16,0)$

$30x + 15y = 300$ *has intercepts* $(0, 20)\&\ (10,0)$

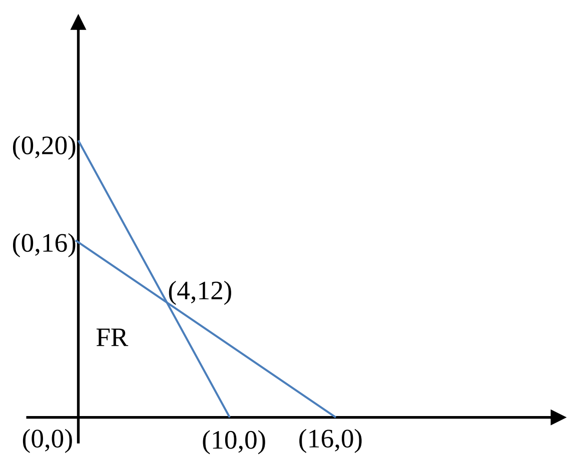
$x + y = 16 \implies y = 16 - x$

$30x + 15(16 - x) = 300$

$30x + 240 - 15x = 300$

$15x = 60$

$x = 4, y = 16 - 4 = 12$



Vertices (list all): $(\mathbf{x}, \mathbf{y}) = (\mathbf{0}, \mathbf{0}), (\mathbf{0}, \mathbf{16}), (\mathbf{10}, \mathbf{0}), (\mathbf{4}, \mathbf{12})$

d) (2 pts) Max possible increase in GPA is **1.68**

$G(0,0) = 0,\ G(0,16) = 1.6,\ G(10,0) = 1.2,\ G(4,12) = 1.68$