MATH 112 B

Sample Exam II - Solutions

1. (a) Solve $x - 2y = -3$ for x: $x = 2y - 3$. Plug into $5x - 4y = 9$: $5(3 - 2y) - 4y = 9$. Solve for y: $y = 4$. Plug into $x = 2y - 3$: $x = 2(4) - 3 = -5$.

   The point of intersection is $(5, 4)$.

   (b) Solve $2x - y = 6$ for y: $y = 2x - 6$. Plug into $5x - 3y = 16$: $5x - 3(2x - 6) = 16$. Solve for x: $x = 2$. Plug into $y = 2x - 6$: $y = 2(2) - 6 = -2$.

   The point of intersection is $(2, -2)$.

2. $f'(x) = x^4 - 7x^3 + 10x^2$ and $f''(x) = 4x^3 - 21x^2 + 20x$. $f'(5) = 0$ and $f''(5) = 75$. Since $f'(5) = 0$, $f(x)$ has a horizontal tangent line at $x = 5$. Since $f''(5) > 0$, $f(x)$ is concave up at $x = 5$. By the second derivative test, $f(x)$ has a local minimum at $x = 5$. (NOTE: Checking that $f'(5) = 0$ is a necessary and important part of the second derivative test even though it does not involve the second derivative.)

3. $\dfrac{\partial E}{\partial b} = 2b + 46m - 35.87$; $\dfrac{\partial E}{\partial m} = 466m + 46b - 201.4$; $y = 1.0534x - 6.2931$.

4. (a) $\sum y_i^2 = 242$

   (b) $\sum x_i y_i = 243$

5. (a) $C'(q) = \dfrac{1}{20}q^2 - \dfrac{6}{5}q + \dfrac{27}{5}$. We set this equal to 0 and multiply by 20 to clear the denominators: $q^2 - 24q + 108 = 0$. Either factor or use the quadratic formula to find that $q = 6$ and $q = 18$ are the values at which $C(q)$ has a horizontal tangent.

   (b) $C''(q) = \dfrac{1}{10}q - \dfrac{6}{5}$. In particular, $C''(6) = -0.6 < 0$. So, $C(q)$ is concave down at $q = 6$. Since $C(q)$ has a horizontal tangent and is concave down at $q = 6$, $C(q)$ has a local maximum there. Similarly $C''(18) = 0.6 > 0$. So, $C(q)$ is concave up at $q = 6$. Since $C(q)$ has a horizontal tangent and is concave up at $q = 18$, $C(q)$ has a local minimum there.

   (c) We want to find the global maximum value of $C(q)$ on the interval $x = 4$ to $x = 25$. We plug our endpoints and the values from (a) into $C(q)$: $C(4) = 28.1$, $C(25) = 35.4$, $C(6) = 29.4$, and $C(18) = 5$. The global maximum is 35.4. The quantity that produces the global maximum is $q = 25$.

6. (a) $P_L(L, K) = 1.5(0.4)L^{-0.6}K^{0.5} + 0.3 = 0.6L^{-0.6}K^{0.5} + 0.3$

   $P_K(L, K) = 1.5(0.5)L^{0.4}K^{-0.5} + 0.2 = 0.75L^{0.4}K^{-0.5} + 0.2$

   (b) We plug $L = 12$ and $K = 10$ into the partial derivatives found in part (a). $P_L(12, 10) = 0.7272$. This is the rate of change of production if we hold K constant and allow L to vary. $P_K(12, 10) = 0.8408$. This is the rate of change of production if we hold L constant and allow K to vary. Since the rate of change of production is higher if we allow K to vary, we should keep L where it is and invest the \$1000 in capital.

7. (a) $P(x, y) = 14x + 6y$

(b) $m(x, y) = 2x + 4y$

(c) $r(x, y) = 3x + y$

(d) To graph the feasible region, you'll need to graph the lines $2x + 4y = 148$ and $3x + y = 92$. The line $2x + 4y = 148$ has x-intercept (74,0) and y-intercept (0,37). The line $3x + y = 92$ has x-intercept (30.7,0) and y-intercept (0,92). The two lines intersect at the point (22,26). This creates a feasible region with vertices at the points (0,0), (30.7,0), (0,37), and (22,26).

(e) To determine maximum profit, we plug each vertex of the feasible region into $P(x, y)$:

$P(0, 0) = \$0$, $P(30.7, 0) = \$429.80$, $P(0, 37) = \$222$, $P(22, 26) = \$464$.

So, the group should sell 22 Math-o-grams and 26 Pie-o-grams in order to maximize profit.

(f) Maximum profit is \$464.