# Math 112 - Winter 2006
# Exam 2
# February 23, 2006

Name: Instructor's Solutions

Section: Version 1

Student ID Number: ______________________

TA's Name: ______________________

| | | |
|---|---|---|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 10 | |
| Total | 50 | |

- Check that your exam contains all the problems listed above.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit.
- You may use a calculator for arithmetic purposes only (such as plugging into the quadratic formula or plugging into a function). ALL other work must be written and demonstrated on your exam. No credit will be given for guess and check or calculator methods, even if they give the correct answer.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- There are multiple versions of the exam. Any student found engaging in academic misconduct will receive a score of 0 on this exam.

GOOD LUCK!

1. (10 points) Compute the following derivatives and partial derivatives as indicated. The correct answer with no supporting work receives *no points.* You do not have to simplify your final answer.

(a) (5 points) Let $g(x) = \frac{5\ln(x)+1}{x^2}$. Find $g'(x)$.

$\left(\frac{N}{D}\right)' = \frac{DN' - ND'}{D^2}$ +3

+1 +1

$$\frac{x^2 \, 5\frac{1}{x} - (5\ln(x)+1)\,2x}{(x^2)^2}$$

don't have to simplify

$$\frac{5x - 10x\ln(x) - 2x}{x^4}$$

+3 using a rule correctly (could also use product rule)

ANSWER: $g'(x) =$ $\frac{3 - 10\ln(x)}{x^3}$

(b) (5 points) Let $f(x,y) = x^2e^{2x} + y^3x$. Find $f_x(x,y)$ and $f_y(x,y)$.

$(FS)' = FS' + SF'$ ⟵ +2

$$f_x(x,y) = x^2 \underbrace{e^{2x} \cdot 2}_{+1} + e^{2x}\,2x + \underbrace{y^3}_{+1}$$

$$f_y(x,y) = \underbrace{3y^2x}_{+1}$$

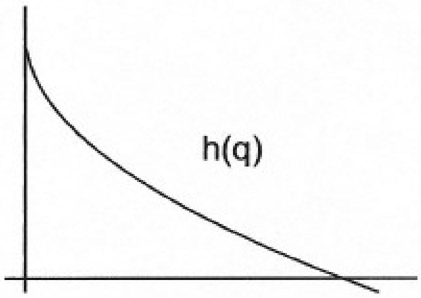
ANSWER: $f_x(x,y) =$ $2x^2e^{2x} + 2xe^{2x} + y^3$ $f_y(x,y) =$ $3y^2x$

2. (10 points)

You own a company that sells Blivets. The demand curve of Blivets is given by the formula

$$p = h(q) = 180 - 20\sqrt{q},$$

where $q$ is measured in Blivets and $p$ is measured in dollars. The graph of the demand curve is given at right.




(a) (3 points) Find the largest interval with $q \geq 0$ on which the demand function is positive and decreasing.

$$180 - 20\sqrt{q} = 0$$
$$\sqrt{q} = 9$$
$$q = 81$$

ANSWER: from $q = 0$ to $q =$ 81

(b) (7 points) Find the price that corresponds to the quantity that gives the largest possible value of total revenue.

$TR(q) = 180q - 20q^{1.5}$ +1

$TR'(q) = 180 - 30q^{0.5}$ + 2

$180 - 30\sqrt{q} = 0$

$\sqrt{q} = 6$ + 2

$q = 36$

price $= h(36) = 180 - 20\sqrt{36}$
$= 60$ +1

$TR(0) = 0$

$TR(36) = 60 \times 36$ +1

$TR(81) = 0$

ANSWER: price = 60 dollars

3. (10 points) Consider the function

$$f(x) = 140\ln(x) + x^2 - 47x + 120.$$

(a) (5 points) Find all critical numbers of $f(x)$.

+2 $f'(x) = \frac{140}{x} + 2x - 47 \overset{?}{=} 0$

+2 $2x^2 - 47x + 140 = 0$

$x = \frac{47 \pm \sqrt{47^2 - 4(2)(140)}}{2(2)}$

+1 $= \boxed{3.5 \text{ or } 20}$

ANSWER: list of critical numbers: $x =$ 3.5, 20

(b) (5 points) Use the second derivative to determine whether each of the critical numbers of $f(x)$ give a local minimum, local maximum, or neither. Clearly indicate your answers. (If you classify a critical number without using the second derivative and showing your work, you will receive no credit.)

+1 $f''(x) = -\frac{140}{x^2} + 2$

+2 $f''(3.5) = -\frac{140}{(3.5)^2} + 2 \approx -9.4286 < 0$ Concave down

$f''(20) = -\frac{140}{(20)^2} + 2 = 1.65 > 0$ Concave up

+2 x = 3.5 gives a local maximum

x = 20 gives a local minimum

4. (10 points) Suppose $p = G(r, s) = 2s^2 - 6rs + 12r$.

(a) (~~[illegible]~~ points) Find the partial derivatives $G_r(r, s)$ and $G_s(r, s)$.

2

+1 +1

ANSWER: $G_r(r, s) =$ $-6s + 12$ $G_s(r, s) =$ $4s - 6r$

(b) (4 points) Find all candidates for local maximum and local minimum of $G(r, s)$.

$$-6s + 12 = 0$$
$$s = 2$$

+2

$$4s - 6r = 0$$
$$8 - 6r = 0$$
$$r = 8/6 = 4/3$$

+2

4

ANSWER: list of candidates: $(r, s) =$ $(4/3, 2)$

(c) (~~[illegible]~~ points) If you fix $r$ to be 3, then $p = G(3, s)$ becomes a function of only one variable, the variable $s$. Find all critical numbers for the function $G(3, s)$.

$$\underbrace{G_s(3, s) = 4\cancel{s} - 18}_{+2} \stackrel{?}{=} 0$$

$$s = \frac{18}{4} = 4.5$$

+2

ANSWER: $s =$ $4.5$

5. (10 points) A company makes two types of chocolate chip cookie mixes: "Chocolate Lite" and "Chocolate Overload".

Each bag of Chocolate Lite contains 10 ounces of chocolate chips and 35 ounces of dough, while each bag of Chocolate Overload contains 20 ounces of chocolate chips and 25 ounces of dough. The profit on each bag of Chocolate Lite is \$1.10, while the profit on each bag of Chocolate Overload is \$0.80

Due to limited supply, your company can stock up to 1000 ounces of chocolate chips and 2555 ounces of cookie dough. Let $x$ denote the number of bags of Chocolate Lite, and let $y$ denote the number of bags of Chocolate Overload.

(a) (3 points) Give the constraints and the objective function for this problem.

| | x | y | |
|---|---|---|---|
| chips | 10 | 20 | $\to 10x+20y$ |
| dough | 35 | 25 | $\to 35x+25y$ |
| profit | 1.10 | 0.80 | $\to 1.10x+0.80y$ |

ANSWER: objective function: $1.10x+0.80y = P(x,y)$ +1

constraints: $10x+20y \le 1000$ +1

$35x+25y \le 2555$ +1

(b) (4 points) Sketch the feasible region, clearly label all vertices.

+2 $\begin{cases} 10x+20y=1000 \\ (0,50) \quad (100,0) \end{cases}$ $\quad 35x+25y=2555$ $\quad (0,102.2) \quad (73,0)$

+2 $\begin{cases} x = 100-2y \end{cases}$

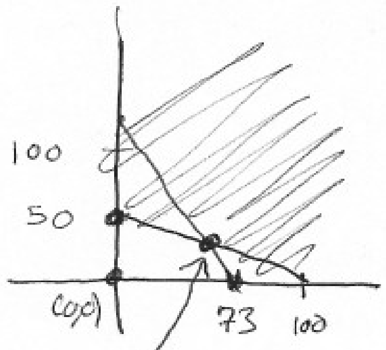
$35(100-2y)+25y=2555$

$3500-70y+25y=2555$

$-45y=-945$

$\boxed{y=21}$

$\boxed{x=58}$




(c) (3 points) How many bags of Chocolate Lite, $x$, and Chocolate Overload, $y$, should the company produce to maximize profit?

+2 $\begin{cases} P(0,0) = \$0 \\ P(73,0) = \$80.30 \\ P(0,50) = \$40 \\ P(58,21) = \$80.60 \end{cases}$

+1

ANSWER: $x =$ 58 bags of Chocolate Lite

$y =$ 21 bags of Chocolate Overload