1 (15 points) Compute the derivatives of the following functions. **Do not simplify**.

a) $f(x) = \sqrt{x^3+1} \cdot e^{x^2+5}$

$$f'(x) = \frac{1}{2}(x^3+1)^{-1/2}(3x^2)e^{x^2+5} + \sqrt{x^3+1}\, e^{x^2+5}(2x)$$

b) $z = \left(1 + ye^y + \frac{1}{y^2}\right)^8$

$$\frac{dz}{dy} = 8\left(1 + ye^y + \frac{1}{y^2}\right)^7 (0 + e^y + ye^y - 2y^{-3})$$

c) $g(t) = \frac{\ln(t^2-t+1)}{3t+5}$

$$g'(t) = \frac{\frac{1}{t^2-t+1}(2t-1)(3t+5) - 3\ln(t^2-t+1)}{(3t+5)^2}$$

2 (8 points)

a) Suppose $g(x,y) = 3x^2 - 5x + 2x^2y - xy^2 + y^3 + 7$. Compute the following partial derivative:

$$g_x(x,y) = 6x - 5 + 4xy - y^2$$

b) Suppose

$$z = \frac{3y}{x^2+1} - xe^y + 2y\ln y.$$

Compute the following partial derivative:

$$\frac{\partial z}{\partial y} = \frac{3}{x^2+1} - xe^y + 2\ln y + 2y\frac{1}{y}$$

3 (5 points) You do **not** know the formula for a certain multi-variable function $f(x,y)$, but you are told that its two partial derivatives are:

$$f_x(x,y) = 2xy + 2y - 5$$

$$f_y(x,y) = x^2 + y - 4$$

Compute or approximate each of the following three values. Show your work.

$$A = \frac{f(1, 3.0001) - f(1,3)}{0.0001} \cong f_y(1,3) = 1 + 3 - 4 = 0$$

$$B = \frac{f(2.001, 3) - f(2,3)}{0.001} \cong f_x(2,3) = 12 + 6 - 5 = 13$$

$C =$ the slope of the tangent line to the graph of $h(x) = f(x,2)$ at $x = 5$.

This equals $f_x(5,2) = 20 + 4 - 5 = 19$

ANSWER: A ≅___0_____, B ≅ ____13____, C = ___19______

4 (13 points) Consider the function $f(t) = t^3 + 3t^2 - 9t + 700$.

a) (2 pts) Compute all values of t at which the graph of $f(t)$ has a horizontal tangent line.

$$f'(t) = 3t^2 + 6t - 9 = 0$$

ANSWER: $t = -3, 1$ (list all)

b) (4 pts) For each of the points you found in part (a), use the second derivative test to determine whether it is a local minimum or local maximum. Show your work and circle your answers.

$$f''(t) = 6t + 6$$

$f''(-3) = -18 + 6 = -12 < 0$, so $t = -3$ is a local maximum

$f''(1) = 6 + 6 = 12 > 0$, so $t = 1$ is a local minimum

c) (i) (2 pts) Is the graph of $f(t)$ concave up or concave-down at $t = 7$? Justify.

$$f''(7) = 6(7) + 6 > 0$$

ANSWER: It's concave-UP

(ii) (2 pts) Is the point $t = 7$ a local minimum, local maximum, or neither for the function $f(t)$? Justify.

ANSWER (circle one): local minimum; local maximum; neither.

because: $t = 7$ is not a critical number ($f'(7) \neq 0$), so it cannot be a local optimum

d) (3 pts) Find the maximum **value** of $f(t)$ on the interval from $t = -3$ to $t = 10$.

$$f(-3) = -27 + 27 + 27 + 700 = 727$$

$$f(1) = 1 + 3 - 9 + 700 = 695$$

$$f(10) = 1000 + 300 - 90 + 700 = 1910$$

ANSWER: Max value of $f(t)$ on the given interval is _____1910____.

5 (9 points) You run a home-business, knitting and selling mittens and socks.

Each pair of mittens takes you 4 hours of work and 0.75 spools of wool to knit, and it sells for $20.

Each pair of socks takes you 2.5 hours and 1 spool of wool to knit, and it sells for $12.

This week you have at most 40 hours to spend knitting and a supply of 15 spools of wool.

Let x be the number of pairs of mittens you produce this week, and y be the number of pairs of socks.

a) (2 pts) Write down the formula for the function $R(x,y)$ that computes the total revenue you would earn from selling x pairs of mittens and y pairs of socks.

$R(x,y) = \mathbf{20x + 12y}$

b) (5 pts) Draw your feasible region, label it "FR", and compute all its vertices.
(list both coordinates -- you may round them to 2 decimal digits)

Constraints:

Time: $4x + 2.5y \leq 40$

y-int: $x = 0, y = \frac{40}{2.5} = 16$

x-int: $y = 0, x = \frac{40}{4} = 10$

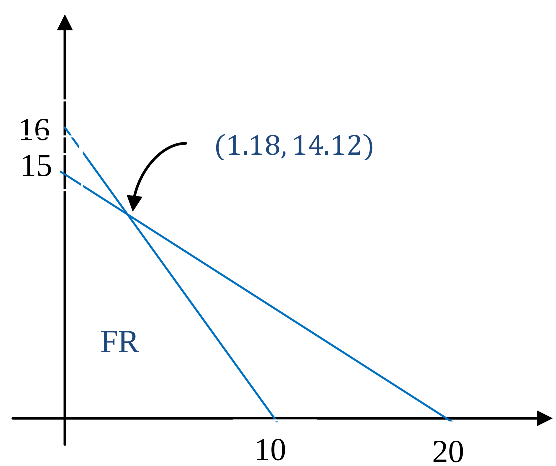
Wool: $0.75x + y \leq 15$

y-int: $x = 0, y = 15$

x-int: $y = 0, x = \frac{15}{0.75} = 20$

Constraint lines intersect at:

$$\begin{cases} 4x + 2.5y = 40 \\ 0.75x + y = 15 \end{cases}$$



From second equation: $y = 15 - 0.75x$. Replace in first: $4x + 2.5(15 - 0.75x) = 40$

Solve for $x = \frac{2.5}{2.125} \cong 1.18$.Compute $y = 15 - 0.75\frac{2.5}{2.125} \cong 14.12$

Vertices (list all): $(x,y) = (0,0), (0,15), (10,0), (1.18, 14.12)$

c) (2 pts) Find your maximum possible total revenue this week.

$R(0,0) = 0$

$R(0,15) = 180$

$R(10,0) = 200$

$R(1.18, 14.12) = 193.04$

Max possible revenue is **$200** (if you sell 10 pairs of mittens and no socks).