MATH 112 B
Final Exam - Version 1
Spring 2002
Hints and Answers

1. (a) HINT: Use the formula for the slope of the secant line with $t = 2$ and $h = 5$.
   ANSWER: 7 feet per second
   (b) HINT: Compute the slope of the tangent line at $t = 5$ by letting h go to 0 in the formula for the slope of the secant line. This gives a formula for instantaneous speed at time t: $s(t) = 3t^2 - 24t + 48$. Plug in 5 for t.
   ANSWER: 3 feet per second
   (c) HINT: You're looking for $d(10) - d(0)$. Use the formula for the slope of the secant line with $t = 0$ and $h = 0$. This will give a formula for $\frac{d(10)-d(0)}{10}$.
   ANSWER: 280 feet

2. (a) ANSWER: $M(q) = \frac{2}{3}q^3 - \frac{35}{2}q^2 + 150q + 92$
   (b) HINT: You need to find the values of q that make $MR' = 0$.
   ANSWER: $q = 7.5, 10$
   (c) HINT: Plug values from part (b) and endpoints into formula for MR. The largest is the global maximum, smallest is the global minimum.
   ANSWER: global maximum value = \$654.50, global minimum value = \$92.00
   (d) HINT: Look at $TR''(9)$.
   ANSWER: The graph of Total Revenue is concave down at $q = 9$ because $TR''(9) = -3 < 0$.

3. (a) ANSWER: $P(x, y) = 1.82x + 1.02y$
   (b) ANSWER: $O(x, y) = 0.80x + 0.55y$, $T(x, y) = 0.20x + 0.45y$
   (c) HINT: Compute $O(1800, 5200)$ and subtract from 4400.
   ANSWER: 100 gallons left over

4. The height of the balloon is given by the function

$$H(m) = \int_0^m h(t)\ dt.$$

   Since the area of the stuff above the t-axis is larger than the area of the stuff below the t-axis, that balloon's height stays above its starting height at all times from $t = 0$ to $t = 7$.

5. (a) ANSWER: $\dfrac{dz}{du} = 7(u^2 + 3u + 4)^6(2u + 3)$
   (b) ANSWER: $\dfrac{\partial z}{\partial y} = 2x^3y + 3x^2 + 4x^2e^y - \dfrac{5}{y}$
   (c) ANSWER: $\displaystyle\int 2 - \frac{5}{\sqrt{x}} + \frac{4}{x}\ dx = 2x - 10\sqrt{x} + 4\ln x + C$
   (d) ANSWER: $\displaystyle\int_0^{12} \frac{1}{3}z^3 - 4z^2 + 12z\ dz = 288$

6. (a) HINT: Anti-differentiate the formula for MR to get a formula for TR:

$$R(q) = -\frac{1}{3}q^3 + \frac{13}{2}q^2 + 23.5q + K.$$

   Since $R(0) = 0$, $K = 0$. Compute $R(6)$.
   ANSWER: 303,000 dollars

(b) HINT: Anti-differentiate the formula for MC to get a formula for TC:

$$C(q) = \frac{1}{3}q^3 - \frac{13}{2}q^2 + 58q + K.$$

Use the fact that $C(0) = 6$ to find the value of K. Divide $C(q)$ by q to get the formula for AC:

$$AC(q) = \frac{1}{3}q^2 - \frac{13}{2}q + 58 + \frac{6}{q}.$$

Plug in $q = 3$.

ANSWER: 43.50 dollars

(c) ANSWER: $-\frac{1}{3}q^3 + \frac{13}{2}q^2 + 23.5q = \frac{1}{3}q^3 - \frac{13}{2}q^2 + 58q + 6$ (or any equivalent form of this equation)

(d) HINT: Look at the graphs of MR and MC. $MR < MC$ from $q = 0$ to $q = 1.5$. This means profit is decreasing on that interval. From $q = 1.5$ to $q = 11.5$, $MR > MC$. This means that profit is increasing on that interval. From $q = 11.5$ on, $MR < MC$ again. So, profit must decrease from $q = 11.5$ on.

ANSWER: (iv)

7. (a) HINT: We know that the derived graph of A is simply the graph of f. $A(m)$ will have horizontal tangents wherever the graph of f touches the x-axis.

ANSWER: $m = 2, 4, 6, 8$

(b) HINT: $A(m)$ will have a local maximum whenever the graph of A changes from increasing to decreasing. This will happen whenever f changes from positive to negative. The graph of A may also have a local maximum at the endpoints of the interval. In this case, $A(0) = 0$ and then A decreases until $x = 2$. So, A has a local max at 0. Also, A is increasing from $x = 8$ to $x = 13$. So, A also has a local max at 13.

ANSWER: $m = 0, 6, 13$

(c) ANSWER: ii. The graph of $f(x)$ and the derived graph of $A(m)$ are the same.

(d) ANSWER: As m increases from 7 to 10, $A(m)$ decreases then increases.

(e) HINT: $A'(3) = f(3) = 2$ (from the graph). $A(3) = \int_0^3 f(x)\,dx$ is the area of the triangle above the x-axis from $x = 2$ to $x = 3$ minus the area of the triangle below the x-axis from $x = 0$ to $x = 2$: $A(3) = -2$.

ANSWER: $A'(3) + A(3) = 0$