MATH 112 A
Final Exam
June 9, 2003

Name _____

Student ID #_____

Section _____

1	15	
2	20	
3	20	
4	12	
5	16	
6	17	
Total	100	

- Check that you have a complete exam. There are six problems, one on each page.
- There are multiple versions of the exam. It will be apparent if you copy someone else's work. Students found engaging in academic misconduct will receive a 0 on this exam.
- You are allowed to use a calculator, a ruler, and one sheet of handwritten notes.
- When rounding is necessary, you may round your final answer to 2 digits after the decimal.
- We can only give you credit for computations that appear on your exam. Show **all** your work.
- If you use a trial and error method when an algebraic method is available, you will not receive full credit.
- Write your answers in the specified locations.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so. If you still need more paper, please ask for some.
- Raise your hand if you have a question.
- You have 1 hour and 50 minutes to complete the exam.

GOOD LUCK!

1. (15 points) Compute each of the following.

(a) $\int 5x - \frac{3}{x^7} + 2\sqrt{x}\ dx$

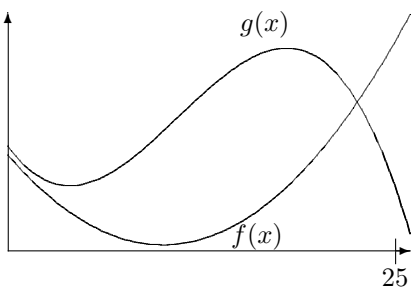
(b) $\int_1^5 -x^2 + 6x - 5\ dx$

(c) $\frac{dy}{dx}$ if $y = \frac{\sqrt{x} \cdot \ln x}{e^{2x}}$ (Do not simplify.)

2. (20 points) To the right are rough sketches of the graphs of two functions:

$f(x) = 3x^2 - 60x + 320$ and

$g(x) = -\frac{1}{3}x^3 + 11x^2 - 72x + 350.$



(a) Find the value of x at which the slope of the tangent to $f(x)$ is 75.

ANSWER: $x =$ _____

(b) What is the maximum value of $g'(x)$?

ANSWER: _____

(c) Let $P(x) = g(x) - f(x)$. Find the value of x in the interval from $x = 3$ to $x = 22$ at which $P(x)$ reaches a global minimum.

ANSWER: $x =$_____

(d) Determine whether the graph of $P(x)$ is concave up or concave down at $x = 10$ Show some work that demonstrates how you arrive at your conclusion.

ANSWER: (circle one) concave up concave down

3. (20 points) Jo and Bo are racing their bicycles. After t seconds, Jo is $J(t)$ feet from the starting line and Bo is $B(t)$ feet from the starting line. Jo's instantaneous speed (in feet per second) is given by the function

$$j(t) = -\frac{1}{12}t^2 + 2t + 18.$$

Bo's average speed from t to $t+h$ is given by

$$\frac{B(t+h) - B(t)}{h} = \frac{1}{3}t^2 + \frac{1}{3}th + \frac{1}{9}h^2 - 6t - 3h + 49.$$

Assume $J(0) = 0$.

(a) How far from the starting line is Jo after 8 seconds?

ANSWER: _____feet

(b) Find Jo's average trip speed at $t = 5$ seconds.

ANSWER: _____feet per second

(c) What is Bo's average speed from 3 to 10 seconds?

ANSWER: _____feet per second

(d) Find the formula for $b(t)$, Bo's instantaneous speed at time t.

ANSWER: $b(t) =$_____

(e) At the start of the race, Jo gave Bo a head start so that Bo started 20 feet ahead of Jo. Find the formula for $B(t)$, Bo's distance from the starting line at time t.

ANSWER: $B(t) =$_____

4. (12 points) Fred's Small Appliances, Inc. makes toasters and waffle irons. Let t be the number (measured in thousands) of toasters Fred produces and sells each month and let w be the number (measured in thousands) of waffle irons Fred produces and sells each month.

   (a) Each toaster sells for \$36 and each waffle iron sells for \$42. Find the formula for $R(t, w)$, Fred's monthly Total Revenue (in thousands of dollars).

   ANSWER: $R(t, w) =$_____

   (b) Fred's monthly Total Cost (in thousands of dollars) is given by the formula

   $$C(t, w) = t^2 + 3tw + 1.5w^2 + 20.$$

   Find the formula for $P(t, w)$, Fred's monthly Profit (again in thousands of dollars).

   ANSWER: $P(t, w) =$_____

   (c) Compute the partial derivatives $\dfrac{\partial P}{\partial t}$ and $\dfrac{\partial P}{\partial w}$.

   ANSWER: $\dfrac{\partial P}{\partial t} =$_____

   $\dfrac{\partial P}{\partial w} =$_____

   (d) Suppose Fred produces exactly $t = 1$ thousand toasters per month. How many waffle irons must Fred produce in order to maximize Profit?

   ANSWER: _____thousand waffle irons

5. (16 points) You sell things. The price per Thing is given by the demand function $p = h(q) = \frac{1}{5}q^2 - 4q + 20$, where q is measured in thousands of Things and $p = h(q)$ is measured in dollars per Thing.

   Recall that Total Revenue (in thousands of dollars) is given by $TR(q) = q \cdot h(q)$.

   The formula for Marginal Cost is given by $MC(q) = \frac{3}{40}q^2 + \frac{3}{8}$.

   Total Cost at $q = 8$ is 17.3 thousand dollars.

   (a) Find the formula for Marginal Revenue.

   ANSWER: $MR(q) =$_____

   (b) Other than $q = 0$, for what quantity is Total Revenue equal to 0?

   ANSWER: $q =$_____thousand Things

   (c) Find the formula for Total Cost (in thousands of dollars).

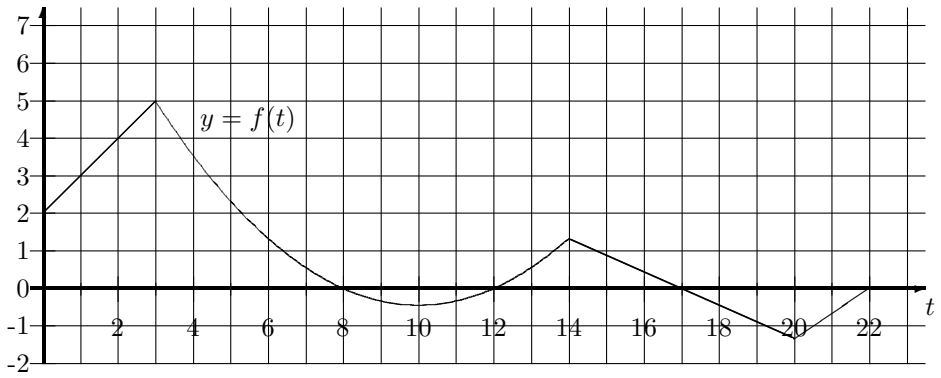
   ANSWER: $TC(q) =$_____

   (d) Write down an equation that you would solve in order to find the quantity at which the tangent line to the graph of Profit has slope 5. Write your equation in the form $(\quad)q^3 + (\quad)q^2 + (\quad)q + (\quad) = 0$. Do not solve the equation.

   ANSWER: $(\qquad)q^3 + (\qquad)q^2 + (\qquad)q + (\qquad) = 0$

6. (17 points) The graph below is of the function $y = f(t)$. Using it, we define another function:

$$A(m) = \int_0^m f(t)\, dt.$$



(a) Compute the approximate value of $A(5)$.

ANSWER: $A(5) =$_____

(b) For how many values of m is $A'(m) = 1$?

ANSWER: _____values of m

(c) Compute the value of $f'(2)$.

ANSWER: $f'(2) =$_____

(d) Name all values of m at which $A(m)$ has a horizontal tangent.

ANSWER: $m =$_____

(e) Indicate whether each of the following statements is True or False.

T **F** $A(14) - A(13)$ is smaller than $A(15) - A(14)$.

T **F** $A(12)$ is smaller than $A(8)$.

T **F** $A(m)$ has a local maximum at $m = 22$.

T **F** On the interval from $m = 0$ to $m = 22$, the global maximum of $A(m)$ occurs at $m = 3$.

T **F** $f(t)$ is decreasing from $t = 3$ to $t = 8$.