MATH 112A
Final Exam Hints and Answers
June 9, 2003

1. (a) ANSWER: $\frac{5}{2}x^2 + \frac{1}{2}x^{-6} + \frac{4}{3}x^{3/2} + K$
   (b) ANSWER: $-\frac{124}{3} + 52 = 10.67$
   (c) ANSWER: $\dfrac{e^{2x}\left[\sqrt{x}\cdot\frac{1}{x} + \ln x \cdot \frac{1}{2\sqrt{x}}\right] - \sqrt{x}\cdot \ln x \cdot e^{2x}\cdot 2}{\left(e^{2x}\right)^2}$

2. (a) HINT: $f'(x) = 6x - 60$. Set this equal to 75 and solve for $x$.
   ANSWER: $x = \frac{135}{6} = 22.5$
   (b) HINT: $g'(x) = -x^2 + 22x - 72$. This is a quadratic whose graph is a parabola that opens downward. Its maximum value is the $y$-coordinate of its vertex: $(11, 49)$.
   ANSWER: 49
   (c) HINT: $P(x) = -\frac{1}{3}x^3 + 8x^2 - 12x + 30$ and $P'(x) = -x^2 + 16x - 12$. Use the quadratic formula to find that $P'(x) = 0$ at $x = 0.7889$ and $x = 15.211$. Compute $P(2)$, $P(15.211)$, and $P(22)$ and choose the smallest value.
   ANSWER: $x = 3$
   (d) HINT: $P''(10) = -4$, which is less than 0.
   ANSWER: concave down

3. (a) HINT: $J(t) = -\frac{1}{36}t^3 + t^2 + 18t + K$. $J(0) = K$ and $J(0) = 0$. So, $K = 0$ and thus $J(t) = -\frac{1}{36}t^3 + t^2 + 18t$. Compute $J(8)$.
   ANSWER: 193.78 feet
   (b) HINT: Compute $\frac{J(5)}{5}$.
   ANSWER: 22.31 feet per second
   (c) HINT: Take $t = 3$ and $h = 7$ and use the formula for $\dfrac{B(t+h) - B(t)}{h}$.
   ANSWER: 25.44 feet per second
   (d) HINT: Let $h$ go to 0 in the formula for $\dfrac{B(t+h) - B(t)}{h}$. That gives $B'(t)$ or $b(t)$.
   ANSWER: $b(t) = \frac{1}{3}t^2 - 6t + 49$
   (e) HINT: Anti-differentiate Bo's speed function: $B(t) = \frac{1}{9}t^3 - 3t^2 + 49t + K$. $B(0) = K$ and $B(0) = 20$. So, $K = 20$.
   ANSWER: $B(t) = \frac{1}{9}t^3 - 3t^2 + 49t + 20$

4. (a) ANSWER: $R(t, w) = 36t + 42w$
   (b) ANSWER: $P(t, w) = 36t + 42w - t^2 - 3tw - 1.5w^2 - 20$
   (c) ANSWER: $\frac{\partial P}{\partial t} = 36 - 2t - 3w$; $\frac{\partial P}{\partial w} = 42 - 3t - 3w$
   (d) HINT: Let $t = 1$ in the formula for profit. Then you have $P(1, w) = -1.5w^2 + 39w + 15$, which is a quadratic function with variable $w$. The maximum profit will occur at its vertex.
   ANSWER: 13 thousand waffle irons

5. (a) HINT: $TR(q) = p \cdot q = \frac{1}{5}q^3 - 4q^2 + 20$ and $MR(q) = TR'(q)$.
   ANSWER: $MR(q) = \frac{3}{5}q^2 - 8q + 20$
   (b) HINT: $TR(q) = p \cdot q = 0$ if $q = 0$ or if $p = 0$. Use the quadratic formula to find where $p$ is equal to 0.
   ANSWER: $q = 10$ thousand Things
   (c) HINT: $TC(q) = \frac{1}{40}q^3 + \frac{3}{8}q + K$. $TC(8) = 15.8 + K$ and $TC(8) = 17.3$. So, $K = 1.5$.
   ANSWER: $TC(q) = \frac{1}{40}q^3 + \frac{3}{8}q + 1.5$
   (d) HINT: $P(q) = \frac{7}{40}q^3 - 4q^2 + \frac{157}{8}q - 1.5$ and $P'(q) = \frac{21}{40}q^2 - 8q + \frac{157}{8}$. Set $P'(q) = 5$ and put the equation in the desired form.
   ANSWER: $(0)q^3 + \left(\frac{21}{40}\right)q^2 + (-8)q + \left(\frac{117}{8}\right) = 0$

6. (a) HINT: Count squares or use trapezoids.
   ANSWER: We accepted answers between 17 and 18.
   (b) HINT: $A'$ is the same as $f$.
   ANSWER: 3 values of $m$
   (c) HINT: Compute the slope of the tangent line to $f$ at $t = 2$.
   ANSWER: $f'(2) = 1$
   (d) HINT: $A(m)$ has a horizontal tangent whenever the graph of $A'$ (which is the same as $f$) crosses the $t$-axis.
   ANSWER: $m$ =8, 12, 17, 22
   (e) ANSWERS: T T F F T